POUNDING AND IMPACT OF BASE ISOLATED BUILDINGS DUE TO EARTHQUAKES

A Thesis

by

VIVEK KUMAR AGARWAL

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

May 2004

Major Subject: Civil Engineering

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Approved as to style and content by:

John M. Niedzwecki (Chair of Committee)	Ken Reinschmidt (Member)
H. Joseph Newton (Member)	Paul Roschke (Interim Department Head)

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ABSTRACT

Pounding and Impact of Base Isolated Buildings due to Earthquakes. (May 2004)
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Chair of Advisory Committee: Dr. John M. Niedzwecki

As the cost of land in cities increases, the need to build multistory buildings in close proximity to each other also increases. Sometimes, construction materials, other objects and any projections from a building may also decrease the spacing provided between the buildings. This leads to the problem of pounding of these closely placed buildings when responding to earthquake ground motion. The recent advent of base isolation systems and their use as an efficient earthquake force resisting mechanism has led to their increased use in civil engineering structures. At the same time, building codes that reflect best design practice are also evolving.

The movement of these base isolated buildings can also result in building pounding. Since base isolation is itself a relatively new technique, pounding phenomenon in base isolated buildings have not been adequately investigated to date. This study looks at the base isolated response of a single two story building and adjacent two story building systems. Four earthquakes with increasing intensity were used in this study. It was found that it is difficult to anticipate the response of the adjacent buildings due to non-linear behavior of pounding and base isolation. The worst case for pounding was found to occur when a fixed base and base isolated buildings were adjacent to each other.

Dedicated to my mother Prem Lata Gupta and father Suresh Chandra Gupta who have worked hard throughout for my education and gave me the opportunity to come to United States for higher studies.

ACKNOWLEDGMENTS

I would like to take this opportunity to express my deep gratitude to my advisory committee chair Dr. John M. Niedzwecki for giving me the opportunity to work on this research project and his guidance throughout, to its completion. I would also like to thank Dr. Ken Reinschmidt and Dr. H. Joseph Newton for being my committee members and for their helpful suggestions.

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1. INTRODUCTION

Building structures are often built close to each other as in the case of residential building complexes or in downtown of metropolitan cities where the cost of land is high. Due to the close proximity of these structures, they have often been found to impact each other while responding to earthquake induced strong ground motion. An earthquake can cause sudden movement of the ground that is transferred to the structure through foundation. The ground motion during an earthquake is usually defined by a time history of the ground acceleration and can be obtained in three directions by instruments known as strong-motion accelerographs (see for example, Chopra 2001). Evaluating the response of a building structure subjected to earthquake ground motion is a dynamic problem where at any instant, the internal resisting forces of the structure are in equilibrium with the time varying inertia force that is defined as the product of the structural mass and the instantaneous ground acceleration (Clough and Penzien 1993).

To reduce the response of a structure to earthquake excitation, various types of base isolation systems have been proposed. One approach is to modify foundation for a building by introducing a layer of material that has a very low lateral stiffness, thus reducing the natural period of vibration of structure. Another technique that is effective in retrofitting structures is the use of a discrete number of friction bearings located between the foundation and the superstructure. Once installed this allows the structure to slide on its foundation during ground movement. The use of friction bearings reduces the base shear force transferred to the structure and hence reduces the displacement of the floors with respect to the base and the internal forces produced in the structure. They can be designed to permit sliding of the structure only during very strong earthquakes. It is this friction bearing base isolation technique that we will be incorporating in the dynamic model formulated in this research study.

This thesis follows the style and format of the ASCE *Journal of Structural Engineering.*

The differences in geometrical and material properties of adjacent buildings can lead to significant differences in their response to the external forces. Spatial variation of the ground movement in case of an earthquake can also cause the adjacent structures to respond differently. Consequently, adjacent buildings may vibrate quite differently during earthquake and may impact each other at various times during their movement. Due to the huge mass of buildings, the momentum of the vibrating structures is also huge and can cause lot of local damage during an impact. In addition to this, large impact forces can significantly change the response behavior during the periods of impact or building pounding. Anagnostopoulos (1995) gives an account of the various incidents in which damage or collapse of building occurred due to pounding. Pounding phenomenon was also found to occur in multi-frame bridges where the decks of the bridge impacted one another during an earthquake (DesRoches and Muthukumar 2002).

1.1. PREVIOUS WORK ON THE SUBJECT

The simplest way to reduce or avoid pounding is to provide an adequate separation distance between the buildings. The International Building Code (2000) specifies the distance between adjacent buildings to be the square root of the sum of squares of their individual displacements. Kasai *et al* (1996) used the spectral difference (SPD) method to calculate adequate building separation distance. Penzien (1997) studied the non-linear hysteretic response by equivalent linearized single degree of freedom system and then used the complete quadratic mode combination (CQC) method to calculate the building separation. Both SPD and CQC methods account for the phasing associated with vibration of adjacent buildings and give a lower value for the separation distance as compared to that specified by IBC (2000) code. Lin (1997) used the random vibration theory and calculated the mean and standard deviation of the separation distance. Lin and Weng (2001) calculated the probability of seismic pounding between adjacent structures separated by code specified distance and the probability distribution of required separation distance of adjacent buildings to avoid seismic pounding. The dynamic relative response of adjacent buildings was also studied by Westermo (1989) who connected them at a floor level by hinged ended beam to maintain the separation. Later, Hao and Zhang (1999) considered spatial variation of ground acceleration between the two structures and Abdullah *et al* (2001) connected the buildings at roof level by shared tuned mass damper and then compared it when using tuned mass dampers. Valles (1997) introduced the concept of pseudo energy radius to study pounding in terms of energy and calculated the minimum gap to avoid pounding in inelastic structures.

Providing the required separation distance is however not always possible, as the need to place the buildings close to each other due to the economics of the land use or architectural reasons. Also the response of older buildings adjacent to the site needs to be considered. Several models of closely spaced adjacent buildings have been developed. These models can be categorized as lumped mass systems and other models. Lumped mass systems are the most basic idealization of a structure and being relatively straightforward to analyze, are the most popular. Examples of some other models are those developed by Papadrakakis *et al* (1996) and Luco and Barros (1998). Papadrakakis *et al* (1996) developed a three-dimensional model of MDOF system using finite elements. They used the Lagrange Multiplier Method to study the response of two or more adjacent buildings located in series or orthogonal configuration with respect to one another. Luco and Barros (1998) modeled buildings as uniform, elastic, continuous damped shear beams and calculated the optimum value of damping constants of dampers uniformly distributed over the height of the shorter building in order to minimize the peak amplitude at the top of the taller structure. Another method to study the pounding problem is the equivalent static force method in which equivalent static horizontal forces are applied to the building to simulate earthquake. Stavroulakis (1991) used this method and then formed the quadratic minimization problem to solve the impact problem.

The lumped mass models can be further categorized, into models that either include or neglect the consideration of building torsion. For example, Leibovich *et al* (1996) considered rotation of single story adjacent buildings with asymmetric impact and Hao and Shen (2001) investigated two adjacent single story buildings having different eccentricities and considered their coupled torsional-lateral responses. Maison and Kasai (1992) considered three dynamic degrees of freedom per floor (i.e. two lateral translational and one torsional). Since rotation of the buildings is generally quite small when compared to the lateral components, many researchers have neglected the rotation of the building in their models. In these lumped mass models, further variations have been incorporated in order to make the models more realistic or sometimes easy to analyze. These variations have included different methods to simulate impact, the type of foundation (e.g. viscoelastic or fixed) and building behavior as closely coupled system. For example, Anagnostopoulos (1988) modeled adjacent buildings as single degree of freedom lumped mass systems with a spring and dashpot to simulate the impact. Chau and Wei (2001) studied the adjacent buildings as single degree of freedom systems with non-linear Hertzian impacts, taken into account by applying conservation of momentum during impact. Anagnostopoulos and Spiliopoulos (1992) used a multi degree of freedom system with a viscoelastic foundation and a spring and a dashpot at floor levels. Papadrakakis *et al* (1991) provided rocking and translation springs at the base to more accurately model the foundation. They utilized Lagrange multipliers to calculate the time of impact by satisfying the geometric compatibility and then used the impulse-momentum relationship and energy dissipation conditions to calculate velocity after impact. Zhang and Xu (1999) studied the response of two adjacent shear buildings connected to each other at each floor level by viscoelastic dampers represented by Voigt’s model. A survey of earlier research on pounding of buildings has been given in table 1.1. It summarizes chronically the published work of the authors with the method of analysis used, simplified model and earthquakes for which the analysis has been done.

Table 1.1 Survey of earlier research on pounding of buildings

References	Earthquake/Ground excitation	Building model	General Comments
Anagnostopoulos, S. A. (1988). "Pounding of buildings in series during earthquake" *J. Earthquake Eng. Struct. Dyn.*, 16, 443 – 456.	El Centro, Taft, Eureka, Olympia, Parkfield	1. Adjacent buildings as SDOF lumped mass system with bilinear force displacement relationship. 2. Impact is modeled by a spring and a dashpot between the masses and acts when impact occurs.	1. Dynamic equation is solved numerically by Central difference method. 2. Assuming inelastic contact, the impact dashpot constant is calculated using coefficient of restitution.
Westermo, B. D. (1989). "The dynamics of interstructural connection to prevent pounding." *J. Earthquake Eng. Struct. Dyn.*, 18, 687 – 699.	Parkfield and Pacoima dam earthquakes	1. Linear MDOF lumped mass system. 2. Adjacent buildings are connected at the top of the shorter structure by hinge ended beam which maintains separation at that floor.	1. Relative displacement response has been calculated by varying the stiffness of the adjacent structure.
Maison, B. F., and Kasai, K. (1990). "Analysis for type of structural pounding." *J. Struct. Eng.,* 116(4), 957 – 977.	El Centro earthquake	1. Building to be studied is modeled as MDOF system with mass lumped at floor centre of mass. 2. Other building is assumed rigid. 3. Pounding is assumed to occur at a single floor level and at the top of the shorter structure and is simulated by a linear elastic spring which acts when impact occurs.	1. The stiffness matrix of the building has been reduced to three dynamic degrees of freedom per floor. Considers elastic building behavior. 2. Classical damping theory is used and thus damping matrix change during pounding due to change in stiffness matrix due to contribution of impact spring stiffness.

Table 1.1 (continued)

References	Earthquake/Ground excitation	Building model	General Comments
Papadrakakis, M., Mouzakis, H., Plevris, N., and Bitzarar, S. (1991). "A Lagrange multiplier solution method for pounding of buildings during earthquakes." *J. Earthquake Eng. Struct. Dyn.*, 20, 981 – 998.	Harmonic ground motion	1. MDOF lumped mass system with bilinear force displacement relationship and rigid slab. 2. Rocking and translation springs are provided to account for foundation.	1. Lagrange multiplier method is used to satisfy geometric compatibility. 2. Impulse-momentum relationship and energy dissipation conditions are satisfied at impact. 3. Building with floors at different heights is also considered. 4. Pounding is assumed to occur at the top of the shorter building.
Stavroulakis, G. E., and Abdalla, K. M. (1991). "Contact between adjacent structures." *J. Struct. Eng.,* 117(10), 2838 – 2850.	Equivalent static horizontal forces linearly distributed along the height of the structure to simulate the earthquake	1. Two adjacent plane frames with floors at the same level.	1. Quadratic minimization problem have been formulated to minimize potential energy of the structure using Kuhn Tucker optimality conditions to calculate the gap between the structures on application of the loading. 2. Stresses have been post processed using finite element analysis. 3. Gap to restrict the contact forces to acceptable limits have been calculated.
Anagnostopoulos, S. A., and Spiliopoulos, K. V. (1992). "An investigation of earthquake induced pounding between adjacent buildings." *J. Earthquake Eng. Struct. Dyn.*, 21, 289 – 302.	Elcentro, Taft, Eureka, Olympia, Parkfield	1. Adjacent buildings are modeled as MDOF lumped mass system and assumed to be shear beam type with bilinear force – deformation relationship. 2. Impact is modeled by a spring and a dashpot between the masses and acts when impact occurs. 3. The rocking motion is introduced through a viscoelastic foundation modeled by translational and rocking spring dashpots.	1. Dynamic equation is solved numerically by Newmark's method. 2. Building response with pounding and without pounding are compared.

Table 1.1 (continued)

References	Earthquake/Ground excitation	Building model	General Comments
Kasai, K., and Maison, B. F. (1992). "Dynamics of pounding when two buildings collide." *J. Earthquake Eng. Struct. Dyn.*, 21, 771 – 786.	North-South component of the 1940 Elcentro earthquake.	1. Multistory adjacent buildings of different heights with mass lumped at the floor centre of mass. 2. Pounding is assumed to occur at the roof level of the smaller building and is simulated by putting a linear elastic spring which acts when pounding occur.	1. The stiffness matrix of the building has been reduced to three dynamic degrees of freedom per floor. Considers elastic building behavior. 2. Classical damping theory is used and thus damping matrix change during pounding due to change in stiffness matrix due to contribution of impact spring stiffness. 3. Performs dynamic analysis and calculates displacement, drift, shear and overturning moment from pounding.
Jeng, V., Kasai, K., and Maison, B. F. (1992). "A spectral difference method to estimate building separations to avoid pounding." *Earthquake Spectra.,* 8(2), 201 – 223.	Elcentro, Pacoima dam, Taft, Cholame Shandon, 1949 and 1969 Olympia and nine artificial earthquakes.	1. Adjacent buildings are modeled as SDOF systems and consider a straight-line deformed shape.	1. Presents the method called Spectral Difference Method and the Double Difference Combination (DDC) rule based on random vibration theory and calculates the required separation to avoid pounding.
Kasai, K., Jagiasi, A. R., and Jeng, V. (1996). "Inelastic vibration phase theory for seismic pounding mitigation." *J. Struct. Eng.,* 122(10), 1136 – 1146.	Elcentro, Pacoima dam, Taft, Cholame Shandon, Olympia and nine artificial earthquakes.	1. Adjacent buildings are modeled as SDOF systems and consider a straight-line deformed shape.	1. Presents a method called Spectral Difference Method, to calculate the required distance to avoid pounding. It accounts for phasing associated with vibration of adjacent structures.
Leibovich, E., Rutenberg, A., and Yankelevsky, D. Z. (1996). "On eccentric seismic pounding of symmetric buildings." *J. Earthquake Eng. Struct. Dyn.*, 25, 219 – 233.	Elcentro and Bucharest earthquakes.	1. Two single storey adjacent structures. For simulating asymmetric impact, protrusions are considered on one side of the buildings.	1. Takes into account rotation of buildings 2. Inelastic impact modeled by applying coefficient of restitution.

Table 1.1 (continued)

References	Earthquake/Ground excitation	Building model	General Comments
Papadrakakis, M., Apostolopoulou, C., Zacharopoulos, A., and Bitzarakis, S. (1996). "Three-dimensional simulation of structural pounding during earthquakes." *J. Eng. Mech.*, 122(5), 423 – 431.	El Centro and Kalamata earthquake	1. Three-dimensional model of two or more adjacent buildings in series or orthogonal to each other is developed. The structures are modeled as MDOF systems with finite elements and pounding contact can take place between slabs or slabs and columns.	1. Lagrange multiplier method is used to solve the contact-impact problem.
Lin, J. H. (1997). "Separation distance to avoid seismic pounding of adjacent buildings." *J. Earthquake Eng. Struct. Dyn.*, 26, 395 – 403.	Kanai-Tajimi excitations.	1. Models the adjacent buildings as MDOF lumped mass system with impact occurring only at the potential pounding location (usually the top of the smaller building).	1.Calculates the mean and standard deviation of the separation distance of adjacent buildings using random vibration theory to avoid pounding.
Penzien, J. (1997). "Evaluation of building separation distance required to prevent pounding during strong earthquakes." *J. Earthquake Eng. Struct. Dyn.*, 26, 849 – 858.	Normalized acceleration response spectrum in the 1994 Uniform Building Code and corresponding displacement response spectrum.	1. For linear response, two adjacent buildings are modeled as continuous structures. 2. For non-linear hysteretic response, buildings are converted to equivalent linearized SDOF system.	1. Separation distance required between the buildings for no pounding is calculated using the CQC method of weighting normal mode responses.
Valles, R. E., and Reinhorn, A. M. (1997). "Evaluation, prevention and mitigation of pounding effects in building structures." *Report No. NCEER-97-0001*, National Center for Earthquake Engrg. Res., State Univ. of New York, Buffalo, N. Y.	Sinusoidal, Mexico City, Elcentro and Taft earthquakes.	1. Used lumped mass system and simulated the impact by modified Kelvin element.	1. Introduced the concept of pseudo energy radius to study pounding in terms of energy. 2. Calculated minimum gap to avoid pounding, using this method. 3. Studied different mitigation techniques using this method.

Table 1.1 (continued)

References	Earthquake/Ground excitation	Building model	General Comments
Luco, J. E., and Barros, F. C. P. De (1998). "Optimal damping between two adjacent elastic structures." *J. Earthquake Eng. Struct. Dyn.*, 27, 649 – 659.	Filtered version of N-S Elcentro.	1. Buildings modeled as uniform, elastic, continuous and damped shear beams and connected by viscous dampers uniformly distributed over the height of the smaller building.	1. Optimum values for the interconnecting damping constants are determined by minimizing the peak amplitude of the transfer function for the response at the top of the taller structure for the first and second modes of vibration and for different relative heights and masses of the buildings.
Hao, H., and Zhang, S. R. (1999). "Spatial ground motion effect on relative displacement of adjacent building structures." *J. Earthquake Eng. Struct. Dyn.*, 28, 333 – 349.	Filtered Tajimi – Kanai power spectral density function together with an empirical coherency function.	1. Four planar moment resisting frames – 1 storey, 2 storeys, 20 storeys and 24 storeys are considered. Assumes appropriate span length, beam and column dimensions and steel ratio. 2. Lumped mass at each floor is calculated by assuming a unit weight for concrete.	1. Ground excitations at the adjacent supports of two buildings are assumed same. Thus three spatially varying ground displacements are considered. 2. Performs dynamic analysis to get the relative displacement at the top of the smaller building.
Zhang, W. S., and Xu, Y. L. (1999). "Dynamic characteristics and seismic response of adjacent buildings linked by discrete dampers." *J. Earthquake Eng. Struct. Dyn.*, 28, 1163 – 1185.	Pseudo excitation method using Kanai-Tajimi filtered white noise spectrum.	1. Adjacent buildings modeled as lumped mass system and connected to each other at each floor level by viscoelastic dampers represented by Voigt model.	1. Finds the optimal parameters of viscoelastic dampers for achieving maximum seismic response reduction. 2. Performs dynamic analysis and determines the random seismic response of non-classically damped system.

Table 1.1 (continued)

References	Earthquake/Ground excitation	Building model	General Comments
Abdullah, M. M., Hanif, J. H., Richardson, A., and Sobanjo, J. (2001). “Use of a shared tuned mass damper (STMD) to reduce vibration and pounding in adjacent structures.” *J. Earthquake Eng. Struct. Dyn.*, 30, 1185 – 1201.	Elcentro earthquake and Kern County earthquake	1. Equal height adjacent buildings modeled as MDOF lumped mass system with shared tuned mass dampers (STMD) applied at the top of buildings.	1. Performs dynamic analysis and compares the response from STMD with that from TMD’s.
Chau, K. T., and Wei, X. X. (2001). “Pounding of structures modeled as non linear impacts of two oscillators.” *J. Earthquake Eng. Struct. Dyn.*, 30, 633 – 651.	Sine as a function of time.	1. Two SDOF systems having non-linear Hertzian impacts.	1. Performs dynamic analysis to calculate response using Runge-Kutta numerical integration technique. 2. Uses analytical solution for impact velocity for rigid impacts and impact velocity for inelastic impacts by considering coefficient of restitution.
Hao, H., and Shen, J. (2001). “Estimation of relative displacement of two adjacent asymmetric structures.” *J. Earthquake Eng. Struct. Dyn.*, 30, 81 – 96.	Filtered Tajimi-Kanai power spectral density function of ground acceleration.	1. Two square adjacent single storey buildings with different eccentricities. 2. Each structure supported by four identical columns.	1. Relative displacement at two corners of adjacent asymmetric structures by considering their coupled torsional – lateral responses. 2. Maximum relative displacement by standard random vibration method. 3. Both linear elastic response and non linear elastic response considered. 4. Effect of eccentricity, torsional stiffness and ductility ratio for bilinear model and stiffness degrading model studied.

Table 1.1 (continued)

References	Earthquake/Ground excitation	Building model	General Comments
Lin, J. H., and Weng, C. C. (2001). “Probability analysis of seismic pounding of adjacent buildings.” *J. Earthquake Eng. Struct. Dyn.*, 30, 1539 – 1557.	Artificial earthquake motions using design response spectrum of dense soil and soft rock and multiplied by a trapezoidal intensity envelop function to simulate the transient character of real earthquake.	1. Steel moment resisting frame assumed as MDOF lumped mass shear type structural system which exhibits elastoplastic behaviour in the form of a hysterectic restoring force displacement characteristic and excited to non stationary Gaussian random process with zero mean.	1. Pounding is assumed to occur at the top level of the shorter building. Effect of impacts on the response of the structure is neglected. Emphasis is on chance of structural pounding and not on severity or duration of impacts. 2. Investigates the seismic pounding probabilities of adjacent buildings separated by a minimum code specified separation distance and the probability distribution of required separation distance of adjacent buildings to avoid seismic pounding.

1.2. CURRENT STATE OF PRACTICE

1.2.1. FIXED BASE BUILDINGS

International Building Code (IBC) 2000 specifies a spacing between the adjacent buildings equal to the square root of the sum of squares (SRSS) of the individual building displacements. Following is the excerpt of the specification from IBC 2000.

> ***1620.3.6 Building separations.*** *All structures shall be separated from adjoining structures. Separations shall allow for the displacement* δ_M. *Adjacent buildings on the same property shall be separated by at least,* δ_{MT}, *where*
>
> $$\delta_{MT} = \sqrt{(\delta_{M1})^2 + (\delta_{M2})^2} \qquad \textbf{\textit{(Equation 16-66)}}$$
>
> *and* δ_{M1} *and* δ_{M2} *are the displacements of the adjacent buildings.*
>
> *When a structure adjoins a property line not common to a public way, that structure shall also be set back from the property line by at least the displacement,* δ_M, *of that structure.*
>
> ***Exception****: Smaller separations or property line setbacks shall be permitted when justified by rational analyses based on maximum expected ground motions.*

As can be seen, the provision also allows for a smaller spacing in some cases.

1.2.2. BASE ISOLATED BUILDINGS

IBC 2000 specifies the minimum distance between a base isolated building and a fixed obstruction as the total maximum displacement of the base isolated building. Following is the excerpt of the specification for base isolated building from IBC 2000.

> ***1623.5.2.2 Building separations.*** *Minimum separation between the isolated structure and surrounding retaining walls or other fixed obstructions shall not be less than the total maximum displacement.*

The total maximum displacement here also includes the sliding displacement. The specification is not adequate since it does not address the spacing between base isolated and fixed and a base isolated and a base isolated building.

1.3. STUDY OBJECTIVE

Although some of the previous studies accounted for movement of foundation by considering a viscoelastic foundation model that allowed small translations and rotations of foundation during ground motion, they are not able to adequately model sliding friction bearing base isolation systems. The proposed research investigation will address the pounding of structures with friction bearing base isolated systems. This base isolation technique has been found to be very effective in reducing building response behavior as verified analytically and experimentally by Mostaghel and Tanbakuchi (1983), Mostaghel and Khodaverdian (1987), Constantinou *et al* (1990) and Mokha *et al* (1990). Since this base isolation system allows the structure to move with respect its initial ground position i.e. slide on its foundation, chances of pounding concerns may in some situations increase. Moreover, spacing between base isolated buildings may become inadequate when historic restoration and seismic rehabilitation of old fixed base buildings is done using base isolation systems. Thus there is a need to study the effect of base isolation on pounding of buildings as well as of pounding on these base isolated buildings. The research done henceforth is the first step towards achieving this goal. It also deals with behavior of these closely spaced buildings for different earthquakes and uses them to arrive at some general conclusions. In this research investigation, adjacent buildings are modeled as lumped mass systems with impact taking place only at floor levels. Inelastic impacts will be taken into account by utilizing linear elastic spring and dashpots at the floor level where the impact occurs (Anagnostopoulos and Spiliopoulos

1992). Earlier, studies using random vibration theory to calculate peak responses were performed by Lin (1997), Hao and Zhang (1999) and Hao and Shen (2001). In each of these studies the response was assumed to be a Gaussian stationary process with zero mean. But in the case of base isolated structures where the superstructures are designed to slide over their foundation, the time history of the building displacement with respect to ground will require a rethinking of this and other assumptions.

The base isolation for each building is simulated by allowing the building to slide along a horizontal frictional plane at the foundation elevation (Mostaghel and Tanbakuchi 1983). Initially, this study will study the behavior of a single building contrasting its behavior characteristics with various two building configurations. The base isolation system adopted in this study is flat sliding surface type and no centering force acts at any time nor is there a limit on the maximum sliding displacement by the buildings. Friction coefficients between the sliding surfaces has also been assumed constant which otherwise will vary. Inelastic behavior and any rotation of the buildings are neglected. The dynamic response equations written for each building include effects of sliding and the impact forces. The resulting system of second order equation is recast as a system of first order ordinary differential equations and solved using MATLAB 'ode' solvers. The external loading on the structures are actual acceleration time histories from earthquakes in California and Mexico. Dynamic equations are written so that the adjacent structures can be subjected to different ground motions. The formulation allows one to study pounding between base isolated and non-base isolated buildings. In this analysis, buildings are modeled as shear buildings and do not include frame behavior. Damage caused by building impacts has not been taken into account and this can make the model prediction unrealistic.

2. MATHEMATICAL FORMULATION

In this chapter the theoretical formulation for modeling adjacent fixed base buildings, single base isolated buildings and adjacent base isolated buildings is presented. This chapter has been divided into three sections. In the first section adjacent fixed base buildings as presented by Anagnostopoulos 1988 and Anagnostopoulos *et al* 1992 are discussed. In the second section a formulation that introduces friction bearing base isolation for a single building is presented. The third section develops the formulation needed to model base isolated buildings with pounding. There is no standard notation for this type of formulation and the intent here is to provide a consistent notation.

2.1. BASIC BUILDING POUNDING MODEL

2.1.1. SINGLE DEGREE OF FREEDOM SYSTEMS

A special three building case of the original n-building formulation subject to earthquake excitation by Anagnostopoulos (1988) is presented. This captures the essence of the more general response formulation by allowing the examination of both interior and exterior buildings. As shown in figure 2.1, the adjacent buildings have been modeled as single degree of freedom (SDOF) systems with lumped masses m_1, m_2 and m_3. The buildings shown are separated distances $d_{11,21}$ between the first and the second building and $d_{21,31}$ between the second and the third building. The stiffnesses of the three buildings are k_{11}, k_{21} and k_{31} and linear viscous dashpot constants for the buildings are c_{11}, c_{21} and c_{31} respectively. Impact between the three buildings has been modeled by introducing a spring and a linear viscous dashpot between the colliding buildings. The stiffness of the spring between the first two buildings is $s_{11,21}$ and that between the last two buildings is $s_{21,31}$. These elements act only when a collision occurs. The corresponding dashpot constants are $c_{11,21}$ and $c_{21,31}$.

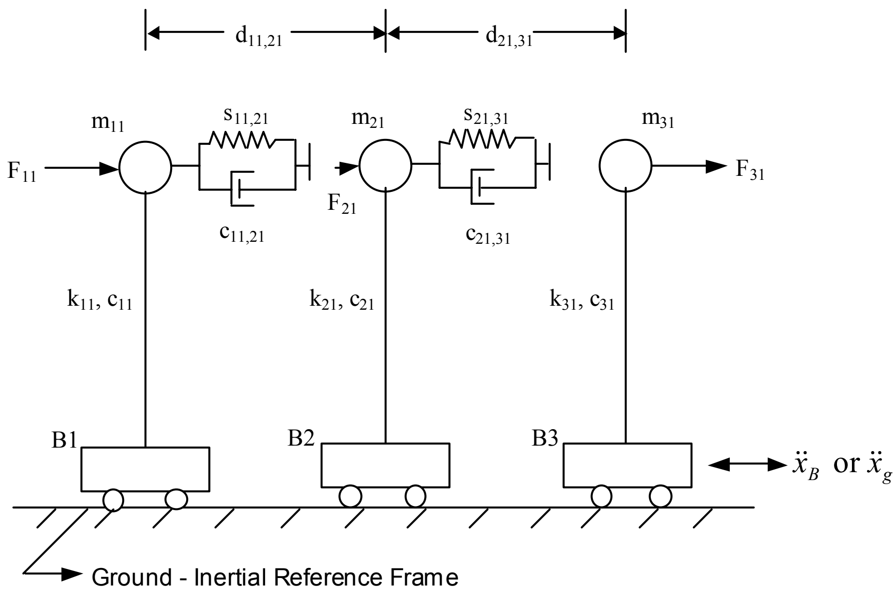

Fig 2.1 Schematic diagram of the three adjacent fixed base buildings

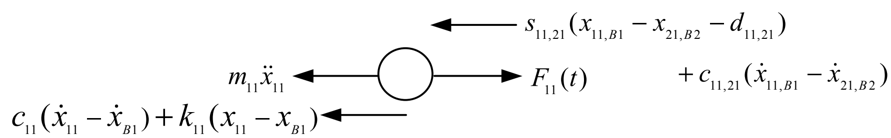

Fig 2.2 Free body diagram for lumped mass m_{11} of first floor of Building 1

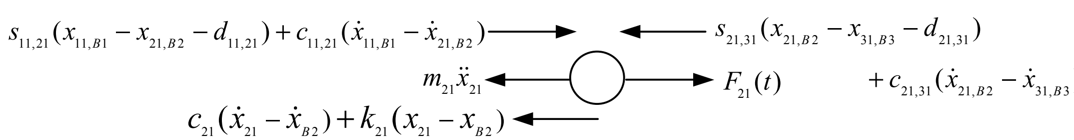

Fig 2.3 Free body diagram for lumped mass m_{21} of first floor of Building 2

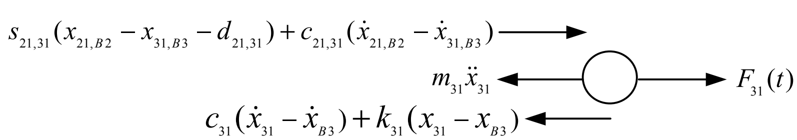

Fig 2.4 Free body diagram for lumped mass m_{31} of first floor of Building 3

The value of these linear viscous dashpot constants is obtained using the coefficient of restitution for impact between the two buildings and thus accounts for inelastic impacts (Anagnostopoulos 1988). The spring constants can be obtained as a function of the stiffness of the colliding buildings.

Dynamic equations for the pounding between single degree of freedom systems, see for example Clough and Penzien (1993), can be written by drawing the free body diagrams for the lumped masses as shown in figures 2.2, 2.3 and 2.4, and then writing the equiliribium equations. The equation of equiliribium of Building 1 that impacts with the second building is

$$\begin{aligned} m_{11}\ddot{x}_{11} + c_{11}(\dot{x}_{11} - \dot{x}_{B1}) + k_{11}(x_{11} - x_{B1}) + s_{11,21}(x_{11,B1} - x_{21,B2} - d_{11,21}) \\ + c_{11,21}(\dot{x}_{11,B1} - \dot{x}_{21,B2}) = F_{11}(t) \end{aligned} \tag{2.1}$$

Here, $\ddot{x}_{11} = \ddot{x}_{11,B1} + \ddot{x}_{B1}$ = absolute acceleration

$\dot{x}_{11} - \dot{x}_{B1} = \dot{x}_{11,B1}$ = relative acceleration with respect to base

$x_{11} - x_{B1} = x_{11,B1}$ = relative displacement with respect to base

Substituting for the above relations in equation (2.1) and rearranging

$$\begin{aligned} m_{11}\ddot{x}_{11,B1} + c_{11}\dot{x}_{11,B1} + k_{11}x_{11,B1} + s_{11,21}(x_{11,B1} - x_{21,B2} - d_{11,21}) \\ + c_{11,21}(\dot{x}_{11,B1} - \dot{x}_{21,B2}) = F_{11}(t) - m_{11}\ddot{x}_{B1} \end{aligned} \tag{2.2}$$

Similarly, for interior building 2, one obtains

$$\begin{aligned} & m_{21}\ddot{x}_{21} + c_{21}(\dot{x}_{21} - \dot{x}_{B2}) + k_{21}(x_{21} - x_{B2}) \\ & -[s_{11,21}(x_{11,B1} - x_{21,B2} - d_{11,21}) + c_{11,21}(\dot{x}_{11,B1} - \dot{x}_{21,B2})] \\ & +[s_{21,31}(x_{21,B2} - x_{31,B3} - d_{21,31}) + c_{21,31}(\dot{x}_{21,B2} - \dot{x}_{31,B3})] = F_{21}(t) \end{aligned} \tag{2.3}$$

Substituting for absolute acceleration and relative velocity and displacement and rearranging, the equation of motion can be rewritten as

$$\begin{aligned}&m_{21}\ddot{x}_{21,B2}+c_{21}\dot{x}_{21,B2}+k_{21}x_{21,B2}\\&-s_{11,21}(x_{11,B1}-x_{21,B2}-d_{11,21})-c_{11,21}(\dot{x}_{11,B1}-\dot{x}_{21,B2})\\&+s_{21,31}(x_{21,B2}-x_{31,B3}-d_{21,31})+c_{21,31}(\dot{x}_{21,B2}-\dot{x}_{31,B3})=F_{21}(t)-m_{21}\ddot{x}_{B2}\end{aligned}\tag{2.4}$$

Finally for Building 3, which is an exterior building, one obtains

$$\begin{aligned}&m_{31}\ddot{x}_{31}+c_{31}(\dot{x}_{31}-\dot{x}_{B3})+k_{31}(x_{31}-x_{B3})\\&-[s_{21,31}(x_{21,B2}-x_{31,B3}-d_{21,31})+c_{21,31}(\dot{x}_{21,B2}-\dot{x}_{31,B3})]=F_{31}(t)\end{aligned}\tag{2.5}$$

And the corresponding equation of motion is

$$\begin{aligned}&m_{31}\ddot{x}_{31,B3}+c_{31}\dot{x}_{31,B3}+k_{31}x_{31,B3}\\&-s_{21,31}(x_{21,B2}-x_{31,B3}-d_{21,31})-c_{21,31}(\dot{x}_{21,B2}-\dot{x}_{31,B3})=F_{31}(t)-m_{31}\ddot{x}_{B3}\end{aligned}\tag{2.6}$$

In equations (2.2), (2.4) and (2.6), x_{11}, x_{21} and x_{31} denotes the absolute displacements of the lumped mass in buildings 1, 2 and 3 respectively. Here, the first subscript denotes the building number and second denotes the nth lumped floor of the building. $x_{11,B1}$, $x_{21,B2}$ and $x_{31,B3}$ in these equations are the relative displacements of floors with respect to the base of Buildings 1, 2 and 3 respectively. $\ddot{x}_{B1}$, $\ddot{x}_{B2}$ and $\ddot{x}_{B3}$ denotes the acceleration of the base or the ground acceleration to which each of the three buildings are subjected. Since we are not considering the spatial variation of the earthquake, these accelerations will be equal. Equations (2.1), (2.4) and (2.6) are coupled due to the impact terms and should be solved simultaneously. These equations can be more conveniently expressed as

$$\begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{21} & 0 \\ 0 & 0 & m_{31} \end{bmatrix}\begin{bmatrix} \ddot{x}_{11,B1} \\ \ddot{x}_{21,B2} \\ \ddot{x}_{31,B3} \end{bmatrix}+\begin{bmatrix} c_{11} & 0 & 0 \\ 0 & c_{21} & 0 \\ 0 & 0 & c_{31} \end{bmatrix}\begin{bmatrix} \dot{x}_{11,B1} \\ \dot{x}_{21,B2} \\ \dot{x}_{31,B3} \end{bmatrix}+\begin{bmatrix} k_{11} & 0 & 0 \\ 0 & k_{21} & 0 \\ 0 & 0 & k_{31} \end{bmatrix}\begin{bmatrix} x_{11,B1} \\ x_{21,B2} \\ x_{31,B3} \end{bmatrix}+$$
$$\begin{bmatrix} s_{11,21}(x_{11,B1}-x_{21,B2}-d_{11,21})+c_{11,21}(\dot{x}_{11,B1}-\dot{x}_{21,B2}) \\ -[s_{11,21}(x_{11,B1}-x_{21,B2}-d_{11,21})+c_{11,21}(\dot{x}_{11,B1}-\dot{x}_{21,B2})]+[s_{21,31}(x_{21,B2}-x_{31,B3}-d_{21,31})+c_{21,31}(\dot{x}_{21,B2}-\dot{x}_{31,B3})] \\ -s_{21,31}(x_{21,B2}-x_{31,B3}-d_{21,31})-c_{21,31}(\dot{x}_{21,B2}-\dot{x}_{31,B3}) \end{bmatrix} \quad (2.7)$$
$$=\begin{bmatrix} F_{11} \\ F_{21} \\ F_{31} \end{bmatrix}-\begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{21} & 0 \\ 0 & 0 & m_{31} \end{bmatrix}\begin{bmatrix} \ddot{x}_{B1} \\ \ddot{x}_{B2} \\ \ddot{x}_{B3} \end{bmatrix}$$

Collecting the stiffness and damping contributions, one obtains the following equation

$$\begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{21} & 0 \\ 0 & 0 & m_{31} \end{bmatrix}\begin{bmatrix} \ddot{x}_{11,B1} \\ \ddot{x}_{21,B2} \\ \ddot{x}_{31,B3} \end{bmatrix}+\left(\begin{bmatrix} c_{11} & 0 & 0 \\ 0 & c_{21} & 0 \\ 0 & 0 & c_{31} \end{bmatrix}+\begin{bmatrix} c_{11,21} & -c_{11,21} & 0 \\ -c_{11,21} & c_{11,21}+c_{21,31} & -c_{21,31} \\ 0 & -c_{21,31} & c_{21,31} \end{bmatrix}\right)\begin{bmatrix} \dot{x}_{11,B1} \\ \dot{x}_{21,B2} \\ \dot{x}_{31,B3} \end{bmatrix}$$
$$+\left(\begin{bmatrix} k_{11} & 0 & 0 \\ 0 & k_{21} & 0 \\ 0 & 0 & k_{31} \end{bmatrix}+\begin{bmatrix} s_{11,21} & -s_{11,21} & 0 \\ -s_{11,21} & s_{11,21}+s_{21,31} & -s_{21,31} \\ 0 & -s_{21,31} & s_{21,31} \end{bmatrix}\right)\begin{bmatrix} x_{11,B1} \\ x_{21,B2} \\ x_{31,B3} \end{bmatrix}+\begin{bmatrix} -s_{11,21}d_{11,21} \\ s_{11,21}d_{11,21}-s_{21,31}d_{21,31} \\ s_{21,31}d_{21,31} \end{bmatrix} \quad (2.8)$$
$$=\begin{bmatrix} F_{11} \\ F_{21} \\ F_{31} \end{bmatrix}-\begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{21} & 0 \\ 0 & 0 & m_{31} \end{bmatrix}\begin{bmatrix} \ddot{x}_{B1} \\ \ddot{x}_{B2} \\ \ddot{x}_{B3} \end{bmatrix}$$

When the response of the three buildings is such that there is no collision between two adjacent buildings, for example at the start of the simulation when the masses are stationary, the impact spring and dashpot do not act and the values assigned to $s_{11,21}$, $s_{21,31}$, $c_{11,21}$ and $c_{21,31}$ should be zero, in the above equation. More formally these constraints can be expressed as

$$\begin{aligned} &x_{11,B1}-x_{21,B2}-d_{11,21}\le 0 \quad \Rightarrow \quad s_{11,21},c_{11,21}=0 \\ &x_{21,B2}-x_{31,B3}-d_{21,31}\le 0 \quad \Rightarrow \quad s_{21,31},c_{21,31}=0 \end{aligned} \quad (2.9)$$

Contact between the lumped masses of floors occur when either $x_{11,B1} - x_{21,B2} - d_{11,21} > 0$ or $x_{21,B2} - x_{31,B3} - d_{21,31} > 0$ are satisfied. When this happens the first building progressively collides with the second. This occurs because the width of the buildings or lumped masses in our model has been taken equal to zero. Clearly, actual buildings have a finite width, and one can interpret this to mean that the first building collides with and perhaps damages the second building. From this perspective the model simulates the actual buildings very well. Using the displacement and the velocity response of the system, impact forces occurring between the buildings can also be calculated. The magnitude of impact force between Building 1 and Building 2 is given by the expression

$$\left| s_{11,21}(x_{11,B1} - x_{21,B2} - d_{11,21}) + c_{11,21}(\dot{x}_{11,B1} - \dot{x}_{21,B2}) \right| \tag{2.10}$$

This stiffness and damping forces act to the left on Building 1 and to right on Building 2. Similarly impact force between Buildings 2 and 3 is given by the expression

$$\left| s_{21,31}(x_{21,B2} - x_{31,B3} - d_{21,31}) + c_{21,31}(\dot{x}_{21,B2} - \dot{x}_{31,B3}) \right| \tag{2.11}$$

This force acts to the left on Building 2 and to the right on Building 3.

2.1.2. TWO DEGREE OF FREEDOM SYSTEMS

In a two degree of freedom model, the impact can take place at both the floor levels. The impact is simulated by providing spring and a dashpot as in the case of single degree of freedom system. Here we consider two buildings, both of which are modeled as two degree of freedom systems as illustrated in figure 2.5. Construction debris or any protrusions between the two buildings may decrease the uniformity of separation between the buildings and hence at different levels may be different. The provision to take different gap has been taken in to account by considering different gap at the two

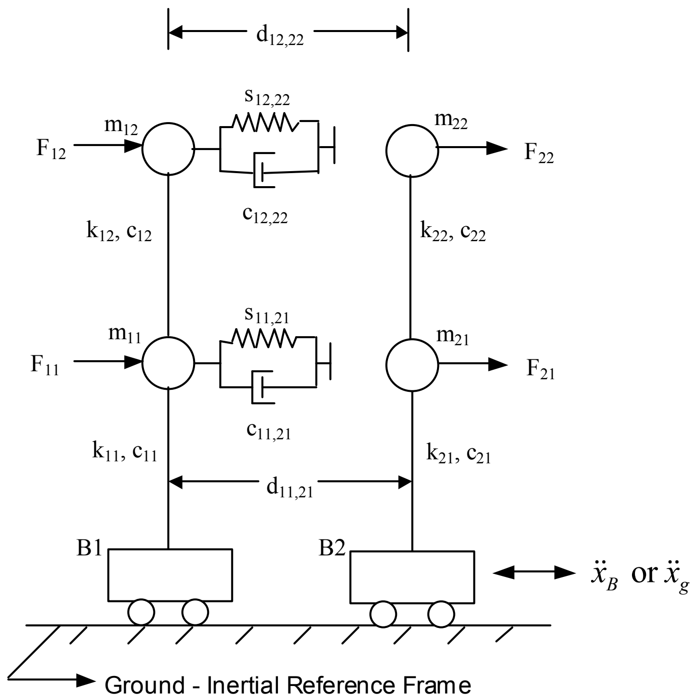

Fig 2.5 Schematic diagram of the two adjacent 2-DOF fixed base systems

levels. The equation of motion for these building models that include the effect of building pounding can be obtained by writing the equiliribium equations from the free body diagram of each of the lumped mass of the building as done for single degree of freedom system.

Thus for each of the buildings we obtain two equations which are coupled and can be written in matrix form. The dynamic equation in matrix form for Building 1 is

$$\begin{bmatrix} m_{11} & 0 \\ 0 & m_{12} \end{bmatrix}\begin{bmatrix} \ddot{x}_{11,B1} \\ \ddot{x}_{12,B1} \end{bmatrix}+\begin{bmatrix} c_{11}+c_{12} & -c_{12} \\ -c_{12} & c_{12} \end{bmatrix}\begin{bmatrix} \dot{x}_{11,B1} \\ \dot{x}_{12,B1} \end{bmatrix}+\begin{bmatrix} k_{11}+k_{12} & -k_{12} \\ -k_{12} & k_{12} \end{bmatrix}\begin{bmatrix} x_{11,B1} \\ x_{12,B1} \end{bmatrix}+ \\ \begin{bmatrix} s_{11,21}(x_{11,B1}-x_{21,B2}-d_{11,21})+c_{11,21}(\dot{x}_{11,B1}-\dot{x}_{21,B2}) \\ s_{12,22}(x_{12,B1}-x_{22,B2}-d_{12,22})+c_{12,22}(\dot{x}_{12,B1}-\dot{x}_{22,B2}) \end{bmatrix}=\begin{bmatrix} F_{11} \\ F_{12} \end{bmatrix}-\begin{bmatrix} m_{11} & 0 \\ 0 & m_{12} \end{bmatrix}\begin{bmatrix} \ddot{x}_{B1} \\ \ddot{x}_{B1} \end{bmatrix} \quad (2.12)$$

The response of Building 2 is governed by the equation

$$\begin{bmatrix} m_{21} & 0 \\ 0 & m_{22} \end{bmatrix}\begin{bmatrix} \ddot{x}_{21,B2} \\ \ddot{x}_{22,B2} \end{bmatrix}+\begin{bmatrix} c_{21}+c_{22} & -c_{22} \\ -c_{22} & c_{22} \end{bmatrix}\begin{bmatrix} \dot{x}_{21,B2} \\ \dot{x}_{22,B2} \end{bmatrix}+\begin{bmatrix} k_{21}+k_{22} & -k_{22} \\ -k_{22} & k_{22} \end{bmatrix}\begin{bmatrix} x_{21,B2} \\ x_{22,B2} \end{bmatrix}- \\ \begin{bmatrix} s_{11,21}(x_{11,B1}-x_{21,B2}-d_{11,21})+c_{11,21}(\dot{x}_{11,B1}-\dot{x}_{21,B2}) \\ s_{12,22}(x_{12,B1}-x_{22,B2}-d_{12,22})+c_{12,22}(\dot{x}_{12,B1}-\dot{x}_{22,B2}) \end{bmatrix}=\begin{bmatrix} F_{21} \\ F_{22} \end{bmatrix}-\begin{bmatrix} m_{21} & 0 \\ 0 & m_{22} \end{bmatrix}\begin{bmatrix} \ddot{x}_{B2} \\ \ddot{x}_{B2} \end{bmatrix} \quad (2.13)$$

Equations (2.12) and (2.13) are also coupled due to impact force terms and should be solved simultaneously. A more convenient matrix form can be developed by first combining these equations that lead to the expression

$$
\begin{bmatrix} m_{11} & 0 & 0 & 0 \\ 0 & m_{12} & 0 & 0 \\ 0 & 0 & m_{21} & 0 \\ 0 & 0 & 0 & m_{22} \end{bmatrix}
\begin{bmatrix} \ddot{x}_{11,B1} \\ \ddot{x}_{12,B1} \\ \ddot{x}_{21,B2} \\ \ddot{x}_{22,B2} \end{bmatrix}
+ \begin{bmatrix} c_{11}+c_{12} & -c_{12} & 0 & 0 \\ -c_{12} & c_{12} & 0 & 0 \\ 0 & 0 & c_{21}+c_{22} & -c_{22} \\ 0 & 0 & -c_{22} & c_{22} \end{bmatrix}
\begin{bmatrix} \dot{x}_{11,B1} \\ \dot{x}_{12,B1} \\ \dot{x}_{21,B2} \\ \dot{x}_{22,B2} \end{bmatrix}
$$

$$
+ \begin{bmatrix} k_{11}+k_{12} & -k_{12} & 0 & 0 \\ -k_{12} & k_{12} & 0 & 0 \\ 0 & 0 & k_{21}+k_{22} & -k_{22} \\ 0 & 0 & -k_{22} & k_{22} \end{bmatrix}
\begin{bmatrix} x_{11,B1} \\ x_{12,B1} \\ x_{21,B2} \\ x_{22,B2} \end{bmatrix}
$$

$$
+ \begin{bmatrix} s_{11,21}(x_{11,B1} - x_{21,B2} - d_{11,21}) + c_{11,21}(\dot{x}_{11,B1} - \dot{x}_{21,B2}) \\ s_{12,22}(x_{12,B1} - x_{22,B2} - d_{12,22}) + c_{12,22}(\dot{x}_{12,B1} - \dot{x}_{22,B2}) \\ -s_{11,21}(x_{11,B1} - x_{21,B2} - d_{11,21}) - c_{11,21}(\dot{x}_{11,B1} - \dot{x}_{21,B2}) \\ -s_{12,22}(x_{12,B1} - x_{22,B2} - d_{12,22}) - c_{12,22}(\dot{x}_{12,B1} - \dot{x}_{22,B2}) \end{bmatrix}
$$

$$
= \begin{bmatrix} F_{11} \\ F_{12} \\ F_{21} \\ F_{22} \end{bmatrix}
- \begin{bmatrix} m_{11} & 0 & 0 & 0 \\ 0 & m_{12} & 0 & 0 \\ 0 & 0 & m_{21} & 0 \\ 0 & 0 & 0 & m_{22} \end{bmatrix}
\begin{bmatrix} \ddot{x}_{B1} \\ \ddot{x}_{B1} \\ \ddot{x}_{B2} \\ \ddot{x}_{B2} \end{bmatrix}
\tag{2.14}
$$

Finally, collecting stiffness and damping contributions, the equation of motion for the building system illustrated in figure 2.5 can be rewritten as

$$
\begin{bmatrix} m_{11} & 0 & 0 & 0 \\ 0 & m_{12} & 0 & 0 \\ 0 & 0 & m_{21} & 0 \\ 0 & 0 & 0 & m_{22} \end{bmatrix}
\begin{bmatrix} \ddot{x}_{11,B1} \\ \ddot{x}_{12,B1} \\ \ddot{x}_{21,B2} \\ \ddot{x}_{22,B2} \end{bmatrix} +
$$

$$
\left(\begin{bmatrix} c_{11}+c_{12} & -c_{12} & 0 & 0 \\ -c_{12} & c_{12} & 0 & 0 \\ 0 & 0 & c_{21}+c_{22} & -c_{22} \\ 0 & 0 & -c_{22} & c_{22} \end{bmatrix} + \begin{bmatrix} c_{11,21} & 0 & -c_{11,21} & 0 \\ 0 & c_{12,22} & 0 & -c_{12,22} \\ -c_{11,21} & 0 & c_{11,21} & 0 \\ 0 & -c_{12,22} & 0 & c_{12,22} \end{bmatrix} \right) \begin{bmatrix} \dot{x}_{11,B1} \\ \dot{x}_{12,B1} \\ \dot{x}_{21,B2} \\ \dot{x}_{22,B2} \end{bmatrix} +
$$

$$
\left(\begin{bmatrix} k_{11}+k_{12} & -k_{12} & 0 & 0 \\ -k_{12} & k_{12} & 0 & 0 \\ 0 & 0 & k_{21}+k_{22} & -k_{22} \\ 0 & 0 & -k_{22} & k_{22} \end{bmatrix} + \begin{bmatrix} s_{11,21} & 0 & -s_{11,21} & 0 \\ 0 & s_{12,22} & 0 & -s_{12,22} \\ -s_{11,21} & 0 & s_{11,21} & 0 \\ 0 & -s_{12,22} & 0 & s_{12,22} \end{bmatrix} \right) \begin{bmatrix} x_{11,B1} \\ x_{12,B1} \\ x_{21,B2} \\ x_{22,B2} \end{bmatrix}
$$

$$
+ \begin{bmatrix} -s_{11,21}d_{11,21} \\ -s_{12,22}d_{12,22} \\ s_{11,21}d_{11,21} \\ s_{12,22}d_{12,22} \end{bmatrix} = \begin{bmatrix} F_{11} \\ F_{12} \\ F_{21} \\ F_{22} \end{bmatrix} - \begin{bmatrix} m_{11} & 0 & 0 & 0 \\ 0 & m_{12} & 0 & 0 \\ 0 & 0 & m_{21} & 0 \\ 0 & 0 & 0 & m_{22} \end{bmatrix} \begin{bmatrix} \ddot{x}_{B1} \\ \ddot{x}_{B1} \\ \ddot{x}_{B2} \\ \ddot{x}_{B2} \end{bmatrix} \tag{2.15}
$$

As in the case of single degree of freedom model, for no impact, the following constraints apply

$$
\begin{aligned} x_{11,B1} - x_{21,B2} - d_{11,21} \le 0 \quad &\Rightarrow \quad s_{11,21}, c_{11,21} = 0 \\ x_{12,B1} - x_{22,B2} - d_{12,22} \le 0 \quad &\Rightarrow \quad s_{12,22}, c_{12,22} = 0 \end{aligned} \tag{2.16}
$$

2.2. BASE ISOLATED BUILDING SYSTEM

2.2.1. SINGLE DEGREE OF FREEDOM SYSTEM

Base isolation system in a building system decreases the inertia force acting on the superstructure and hence the deflections and shear forces (Arya 1984). A sliding base isolation system can be provided by using Teflon (TFE) sliding bearings between the superstructure and its foundation and consists of Teflon-steel interfaces (Mokha *et al*

1990 and Constantinou *et al* 1990). In actual building a sliding base isolation system also consists of a centering device or restoring force (Mokha *et al* 1990) so as to avoid the residual displacement of the structure, however in the present study this consideration is neglected. For the sliding isolation system it will be assumed that a constant coefficient of friction is adequate. In an actual device the static coefficient of friction is different from the kinetic value and both of these vary as a function of bearing pressure and sliding velocity (Mokha *et al* 1990). The coefficient of friction provided should be chosen according to the expected maximum acceleration peaks in the ground motion to achieve the maximum advantage of sliding bearing system (Arya 1984).

Here we will develop the governing equation for the response of a single single-degree of freedom base isolated system. Consider the single single-degree of freedom base isolated lumped mass system illustrated in figure 2.6. Unlike the buildings in section 2.1.1, here the base of the building is separated from the foundation by the sliding system. The upper part of the structural system is allowed to slide with respect to the foundation, which has the same motion as the ground during an earthquake. The structure and its base slide on the lower part as one piece. The sliding coefficient of friction is μ_1. To develop the governing equation for the response, consider the free body diagrams of the lumped mass model of the building and the base of the building as shown in figures 2.7 and 2.8. The equiliribium equation of mass m_{11} can be written as

$$m_{11}\ddot{x}_{11} + c_{11}(\dot{x}_{11} - \dot{x}_{B11}) + k_{11}(x_{11} - x_{B11}) = F_{11}(t) \tag{2.17}$$

Here, $\ddot{x}_{11} = \ddot{x}_{11,B11} + \ddot{x}_{B11}$ = absolute acceleration

$\dot{x}_{11} - \dot{x}_{B11} = \dot{x}_{11,B11}$ = relative acceleration with respect to base

$x_{11} - x_{B11} = x_{11,B11}$ = relative displacement with respect to base

Substituting for the above relations in equation (2.17) and rearranging into to a standard form

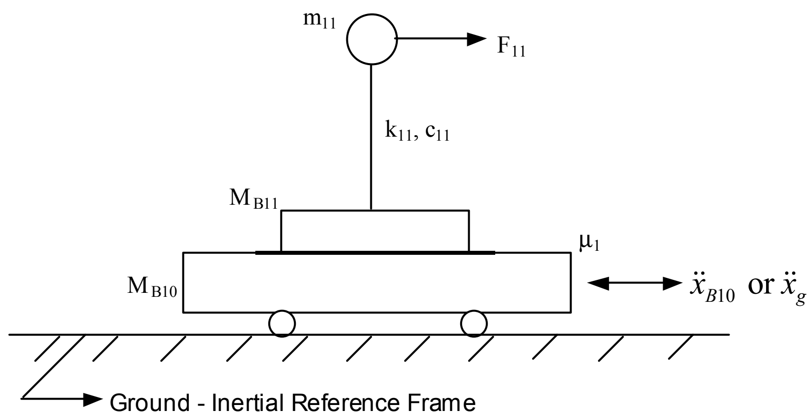

Fig 2.6 Schematic diagram of the single 1-DOF base isolated system

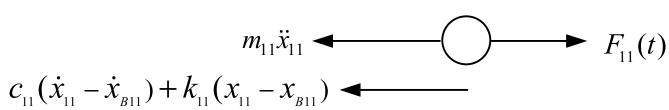

Fig 2.7 Free body diagram for lumped mass m_{11} of the first floor of 1-DOF base isolated system

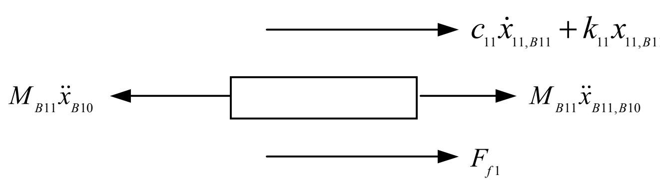

Fig 2.8 Free body diagram for lumped mass M_{B11} of the base of 1-DOF base isolated system

$$
\begin{aligned}
m_{11}\ddot{x}_{11,B11} + c_{11}\dot{x}_{11,B11} + k_{11}x_{11,B11} &= F_{11}(t) - m_{11}\ddot{x}_{B11} \\
&= F_{11}(t) - m_{11}\ddot{x}_{B11,B10} - m_{11}\ddot{x}_{B10}
\end{aligned}
\tag{2.18}
$$

The term $\ddot{x}_{B11,B10}$ in above equation is the sliding acceleration, which is the acceleration of the base of the structure with respect to the foundation. Equiliribium equation for mass M_{B11} is written from its free body diagram in figure 2.8, assuming the ground accelerates to the right hand side at the start of earthquake.

$$
\begin{aligned}
M_{B11}\ddot{x}_{B11,B10} &= c_{11}\dot{x}_{11,B11} + k_{11}x_{11,B11} + F_{f1} - M_{B11}\ddot{x}_{B10} \\
&= F_{11} - m_{11}\ddot{x}_{B11} - m_{11}\ddot{x}_{11,B11} + F_{f1} - M_{B11}\ddot{x}_{B10} \\
&= F_{11} - m_{11}(\ddot{x}_{B11,B10} + \ddot{x}_{B10}) - m_{11}\ddot{x}_{11,B11} + F_{f1} - M_{B11}\ddot{x}_{B10}
\end{aligned}
\tag{2.19}
$$

Simplifying the above equation to obtain

$$
(m_{11} + M_{B11})\ddot{x}_{B11,B10} = F_{f1} + F_{11} - (m_{11} + M_{B11})\ddot{x}_{B10} - m_{11}\ddot{x}_{11,B11} \tag{2.20}
$$

The direction of friction force F_{f1} will depend on the velocity of the base of structure with respect to the foundation and will be in the direction opposite to it. Thus equation (2.20) can be written in general form as

$$
(m_{11} + M_{B11})\ddot{x}_{B11,B10} = -\operatorname{sgn}(\dot{x}_{B11,B10})F_{f1} + F_{11} - (m_{11} + M_{B11})\ddot{x}_{B10} - m_{11}\ddot{x}_{11,B11} \tag{2.21}
$$

Here, $\operatorname{sgn}(x)$ is the signum function and $F_{f1} = \mu_1(m_{11} + M_{B11})g$ is the constant friction force during sliding. Dynamic equation for structure and the equation for sliding are coupled and thus should be solved simultaneously. They are not combined here in a matrix form since dynamic equation of structure is in the standard form that is commonly encountered in books. When there is no sliding, equation (2.21) simply becomes $\ddot{x}_{B11,B10} = 0$.

Sliding occurs when the magnitude of the net force acting on the base of the building is greater then the maximum friction force. At the instant when the sliding starts and during sliding, the following inequality holds

$$\left| M_{B11}\ddot{x}_{B10} + (-c_{11}\dot{x}_{11,B11} - k_{11}x_{11,B11}) \right| > F_{f1} = \mu_1(m_{11} + M_{B11})g \tag{2.22}$$

With some simplifications, it can be rewritten as

$$\left| (m_{11} + M_{B11})\ddot{x}_{B10} + m_{11}\ddot{x}_{B11,B10} + m_{11}\ddot{x}_{11,B11} - F_{11} \right| > \mu_1(m_{11} + M_{B11})g \tag{2.23}$$

The left hand side of (2.23) is the net force on the base and right hand side is the peak value of friction force. This inequality can be used to find whether there is sliding, at any instant. The sliding acceleration $\ddot{x}_{B11,B10}$ in the inequality is non-zero in sliding phase but is zero at the start of sliding.

2.2.2. TWO DEGREE OF FREEDOM SYSTEM

A two-degree of freedom single building model with base isolation is illustrated in figure 2.9. The corresponding free-body diagrams are given in figures 2.10, 2.11 and 2.12. The equiliribium equation for mass m_{12} is

$$m_{12}\ddot{x}_{12} + c_{12}(\dot{x}_{12} - \dot{x}_{11}) + k_{12}(x_{12} - x_{11}) = F_{12}(t) \tag{2.24}$$

With simplifications similar as done before, above equation can be rewritten as

$$\begin{aligned} m_{12}\ddot{x}_{12,B11} + c_{12}\dot{x}_{12,B11} - c_{12}\dot{x}_{11,B11} + k_{12}x_{12,B11} - k_{12}x_{11,B11} \\ = F_{12}(t) - m_{12}\ddot{x}_{B11,B10} - m_{12}\ddot{x}_{B10} \end{aligned} \tag{2.25}$$

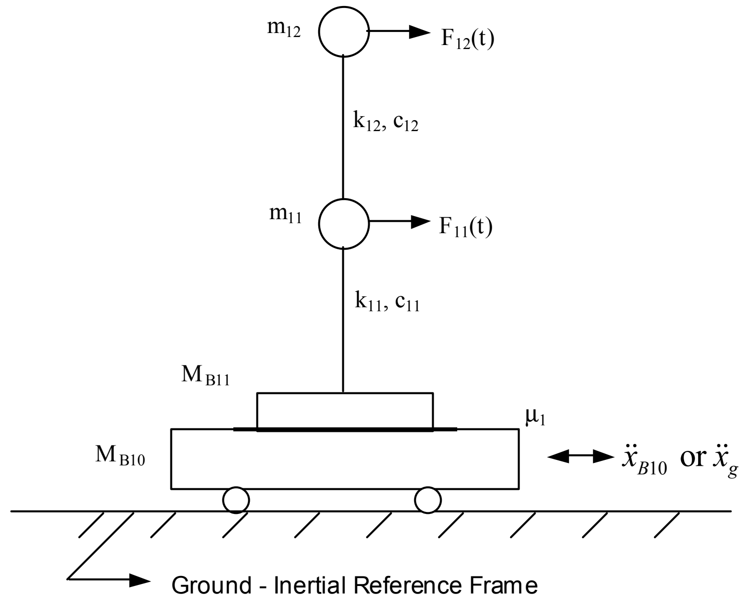

Fig 2.9 Schematic diagram of the single 2-DOF base isolated system

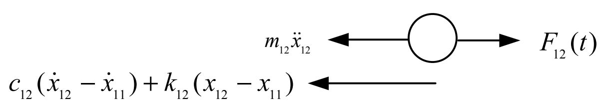

Fig 2.10 Free body diagram for lumped mass m_{12} of second floor

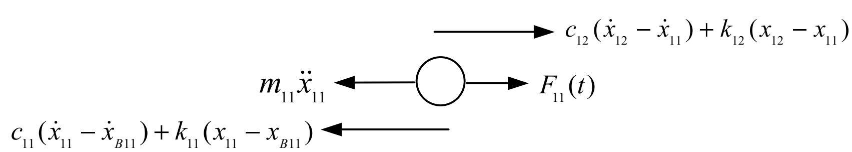

Fig 2.11 Free body diagram for lumped mass m_{11} of first floor

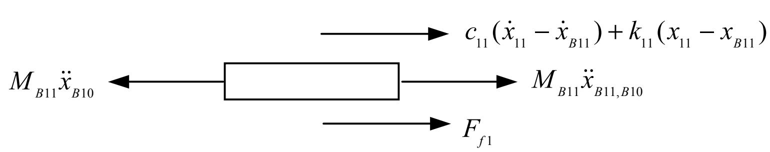

Fig 2.12 Free body diagram for lumped mass M_{B11} of base

Similarly, the equiliribium equation for mass m_{11} is

$$m_{11}\ddot{x}_{11} + c_{11}(\dot{x}_{11} - \dot{x}_{B11}) + k_{11}(x_{11} - x_{B11}) - c_{12}(\dot{x}_{12} - \dot{x}_{11}) - k_{12}(x_{12} - x_{11}) = F_{11}(t) \quad (2.26)$$

On simplification, equation (2.26) is written as

$$\begin{aligned} m_{11}\ddot{x}_{11,B11} + c_{11}\dot{x}_{11,B11} + k_{11}x_{11,B11} - c_{12}\dot{x}_{12,B11} + c_{12}\dot{x}_{11,B11} - k_{12}x_{12,B11} \\ + k_{12}x_{11,B11} = F_{11}(t) - m_{11}\ddot{x}_{B11,B10} - m_{11}\ddot{x}_{B10} \end{aligned} \quad (2.27)$$

Equations (2.25) and (2.27) can be conveniently written as

$$\begin{aligned} \begin{bmatrix} m_{11}\ddot{x}_{11,B11} \\ m_{12}\ddot{x}_{12,B11} \end{bmatrix} + \begin{bmatrix} (c_{11}+c_{12})\dot{x}_{11,B11} - c_{12}\dot{x}_{12,B11} \\ -c_{12}\dot{x}_{11,B11} + c_{12}\dot{x}_{12,B11} \end{bmatrix} + \begin{bmatrix} (k_{11}+k_{12})x_{11,B11} - k_{12}x_{12,B11} \\ -k_{12}x_{11,B11} + k_{12}x_{12,B11} \end{bmatrix} = \begin{bmatrix} F_{11} \\ F_{12} \end{bmatrix} \\ - \begin{bmatrix} m_{11}\ddot{x}_{B11,B10} + m_{11}\ddot{x}_{B10} \\ m_{12}\ddot{x}_{B11,B10} + m_{12}\ddot{x}_{B10} \end{bmatrix} \end{aligned} \quad (2.28)$$

Collecting the stiffness and damping contributions, one obtains the following equation

$$\begin{aligned} \begin{bmatrix} m_{11} & 0 \\ 0 & m_{12} \end{bmatrix} \begin{bmatrix} \ddot{x}_{11,B11} \\ \ddot{x}_{12,B11} \end{bmatrix} + \begin{bmatrix} c_{11}+c_{12} & -c_{12} \\ -c_{12} & c_{12} \end{bmatrix} \begin{bmatrix} \dot{x}_{11,B11} \\ \dot{x}_{12,B11} \end{bmatrix} + \begin{bmatrix} k_{11}+k_{12} & -k_{12} \\ -k_{12} & k_{12} \end{bmatrix} \begin{bmatrix} x_{11,B11} \\ x_{12,B11} \end{bmatrix} = \begin{bmatrix} F_{11} \\ F_{12} \end{bmatrix} \\ - \begin{bmatrix} m_{11} & 0 \\ 0 & m_{12} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \ddot{x}_{B11,B10} - \begin{bmatrix} m_{11} & 0 \\ 0 & m_{12} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \ddot{x}_{B10} \end{aligned} \quad (2.29)$$

Equiliribium equation for the base of the structure as shown in figure 2.12 is

$$M_{B11}\ddot{x}_{B11,B10} = F_{f1} + c_{11}(\dot{x}_{11} - \dot{x}_{B11}) + k_{11}(x_{11} - x_{B11}) - M_{B11}\ddot{x}_{B10} \quad (2.30)$$

Using equations (2.25) and (2.27) to substitute for $c_{11}(\dot{x}_{11}-\dot{x}_{B11})+k_{11}(x_{11}-x_{B11})$ in the above equation and rearranging, we get the equation for sliding acceleration of base of the structure with respect to the foundation.

$$\begin{aligned}(m_{11}+m_{12}+M_{B11})\ddot{x}_{B11,B10} &= F_{f1}+F_{11}+F_{12}-(m_{11}+m_{12}+M_{B11})\ddot{x}_{B10} \\ &\quad -m_{11}\ddot{x}_{11,B11}-m_{12}\ddot{x}_{12,B11}\end{aligned} \tag{2.31}$$

The general equation for sliding can be written by accounting for the direction of friction force F_{f1}. Rewriting the above equation yields

$$\begin{aligned}(m_{11}+m_{12}+M_{B11})\ddot{x}_{B11,B10} &= -\mathrm{sgn}(\dot{x}_{B11,B10})F_{f1}+F_{11}+F_{12} \\ &\quad -(m_{11}+m_{12}+M_{B11})\ddot{x}_{B10}-m_{11}\ddot{x}_{11,B11}-m_{12}\ddot{x}_{12,B11}\end{aligned} \tag{2.32}$$

Where, $F_{f1}=\mu_1(m_{11}+m_{12}+M_{B11})g$ during sliding. Equation (2.32) should be applied only during sliding. For the case where no sliding occurs, the sliding acceleration is of course zero. Furthermore, in this case, right hand side of equation (2.31) can be used to find the actual friction force in the sliding system.

As for the single degree of freedom system, here also sliding will occur when net force on the base exceeds the friction force. Thus for sliding, the following inequality holds

$$\begin{aligned}\left|M_{B11}\ddot{x}_{B10}+(-c_{11}(\dot{x}_{11}-\dot{x}_{B11})-k_{11}(x_{11}-x_{B11}))\right| &> F_{f1} \\ &= \mu_1(m_{11}+m_{12}+M_{B11})g\end{aligned} \tag{2.33}$$

With simplifications as done before, the inequality reduces to

$$\begin{aligned}\left|(m_{11}+m_{12}+M_{B11})\ddot{x}_{B10}+(m_{11}+m_{12})\ddot{x}_{B11,B10}+m_{11}\ddot{x}_{11,B11}+m_{12}\ddot{x}_{12,B11}-F_{11}-F_{12}\right| \\ > \mu_1(m_{11}+m_{12}+M_{B11})g\end{aligned} \tag{2.34}$$

When there is no sliding, sliding velocity and acceleration are zero for that building.

2.3. BASE ISOLATION WITH BUILDING POUNDING

2.3.1. SINGLE DEGREE OF FREEDOM SYSTEMS

In the previous sections, formulations were developed separately for pounding and base isolation. The next objective is to develop the dynamic equation for three single degree of freedom systems that are base isolated and include pounding considerations as depicted in figure 2.13.

The dynamic equation for this building system can be written by including impact force terms in the dynamic equation of the single degree of freedom base isolated system. The impact force terms couples the three single degree of freedom systems which otherwise are uncoupled. Note that the displacements and velocities used to calculate the impact force will be with respect to the lower foundation portion of structure and not the base of structure, since there will be additional displacement due to sliding.

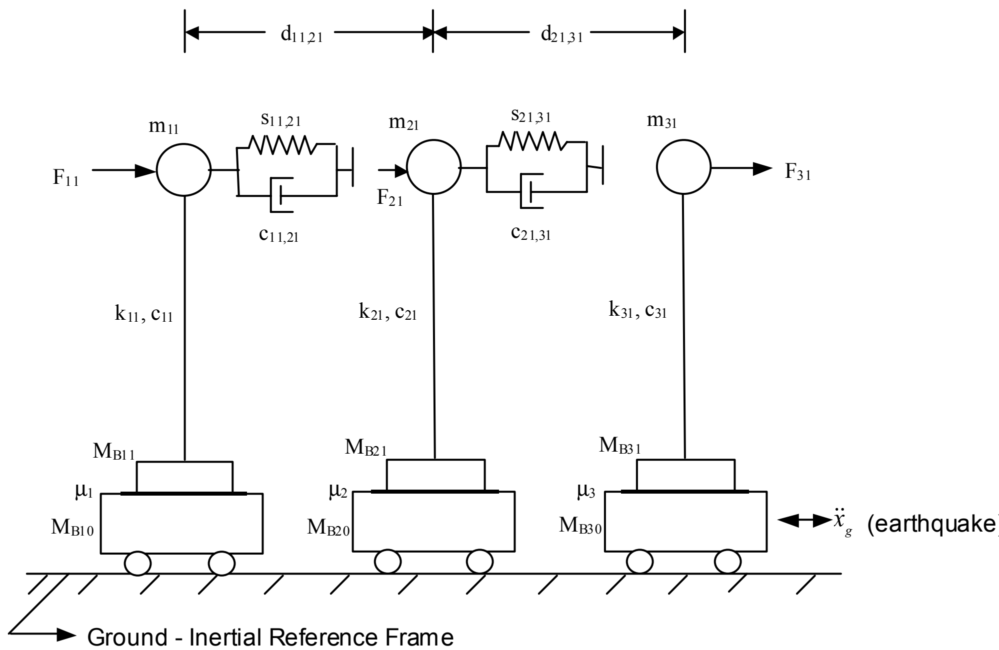

Fig 2.13 Schematic diagram of the three adjacent 1-DOF base isolated systems

The equation of motion is

$$
\begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{21} & 0 \\ 0 & 0 & m_{31} \end{bmatrix}
\begin{bmatrix} \ddot{x}_{11,B11} \\ \ddot{x}_{21,B21} \\ \ddot{x}_{31,B31} \end{bmatrix}
+
\begin{bmatrix} c_{11} & 0 & 0 \\ 0 & c_{21} & 0 \\ 0 & 0 & c_{31} \end{bmatrix}
\begin{bmatrix} \dot{x}_{11,B11} \\ \dot{x}_{21,B21} \\ \dot{x}_{31,B31} \end{bmatrix}
+
\begin{bmatrix} k_{11} & 0 & 0 \\ 0 & k_{21} & 0 \\ 0 & 0 & k_{31} \end{bmatrix}
\begin{bmatrix} x_{11,B11} \\ x_{21,B21} \\ x_{31,B31} \end{bmatrix}
+
$$

$$
\begin{bmatrix}
s_{11,21}(x_{11,B10} - x_{21,B20} - d_{11,21}) + c_{11,21}(\dot{x}_{11,B10} - \dot{x}_{21,B20}) \\
-[s_{11,21}(x_{11,B10} - x_{21,B20} - d_{11,21}) + c_{11,21}(\dot{x}_{11,B10} - \dot{x}_{21,B20})] + [s_{21,31}(x_{21,B20} - x_{31,B30} - d_{21,31}) + c_{21,31}(\dot{x}_{21,B20} - \dot{x}_{31,B30})] \\
-s_{21,31}(x_{21,B20} - x_{31,B30} - d_{21,31}) - c_{21,31}(\dot{x}_{21,B20} - \dot{x}_{31,B30})
\end{bmatrix}
$$

$$
= \begin{bmatrix} F_{11} \\ F_{21} \\ F_{31} \end{bmatrix}
-
\begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{21} & 0 \\ 0 & 0 & m_{31} \end{bmatrix}
\begin{bmatrix} \ddot{x}_{B11,B10} \\ \ddot{x}_{B21,B20} \\ \ddot{x}_{B31,B30} \end{bmatrix}
-
\begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{21} & 0 \\ 0 & 0 & m_{31} \end{bmatrix}
\begin{bmatrix} \ddot{x}_{B10} \\ \ddot{x}_{B20} \\ \ddot{x}_{B30} \end{bmatrix}
\tag{2.35}
$$

Collecting the damping and stiffness contributions, one obtains the following matrix equation

$$
\begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{21} & 0 \\ 0 & 0 & m_{31} \end{bmatrix}
\begin{bmatrix} \ddot{x}_{11,B11} \\ \ddot{x}_{21,B21} \\ \ddot{x}_{31,B31} \end{bmatrix}
+
\left(
\begin{bmatrix} c_{11} & 0 & 0 \\ 0 & c_{21} & 0 \\ 0 & 0 & c_{31} \end{bmatrix}
+
\begin{bmatrix} c_{11,21} & -c_{11,21} & 0 \\ -c_{11,21} & c_{11,21} + c_{21,31} & -c_{21,31} \\ 0 & -c_{21,31} & c_{21,31} \end{bmatrix}
\right)
\begin{bmatrix} \dot{x}_{11,B11} \\ \dot{x}_{21,B21} \\ \dot{x}_{31,B31} \end{bmatrix}
+
$$

$$
\left(
\begin{bmatrix} k_{11} & 0 & 0 \\ 0 & k_{21} & 0 \\ 0 & 0 & k_{31} \end{bmatrix}
+
\begin{bmatrix} s_{11,21} & -s_{11,21} & 0 \\ -s_{11,21} & s_{11,21} + s_{21,31} & -s_{21,31} \\ 0 & -s_{21,31} & s_{21,31} \end{bmatrix}
\right)
\begin{bmatrix} x_{11,B11} \\ x_{21,B21} \\ x_{31,B31} \end{bmatrix}
+
\begin{bmatrix} -s_{11,21}d_{11,21} \\ s_{11,21}d_{11,21} - s_{21,31}d_{21,31} \\ s_{21,31}d_{21,31} \end{bmatrix}
=
\begin{bmatrix} F_{11} \\ F_{21} \\ F_{31} \end{bmatrix}
$$

$$
-
\begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{21} & 0 \\ 0 & 0 & m_{31} \end{bmatrix}
\begin{bmatrix} \ddot{x}_{B11,B10} \\ \ddot{x}_{B21,B20} \\ \ddot{x}_{B31,B30} \end{bmatrix}
-
\begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{21} & 0 \\ 0 & 0 & m_{31} \end{bmatrix}
\begin{bmatrix} \ddot{x}_{B10} \\ \ddot{x}_{B20} \\ \ddot{x}_{B30} \end{bmatrix}
\tag{2.36}
$$

$$
-
\begin{bmatrix} c_{11,21} & -c_{11,21} & 0 \\ -c_{11,21} & c_{11,21} + c_{21,31} & -c_{21,31} \\ 0 & -c_{21,31} & c_{21,31} \end{bmatrix}
\begin{bmatrix} \dot{x}_{B11,B10} \\ \dot{x}_{B21,B20} \\ \dot{x}_{B31,B30} \end{bmatrix}
-
\begin{bmatrix} s_{11,21} & -s_{11,21} & 0 \\ -s_{11,21} & s_{11,21} + s_{21,31} & -s_{21,31} \\ 0 & -s_{21,31} & s_{21,31} \end{bmatrix}
\begin{bmatrix} x_{B11,B10} \\ x_{B21,B20} \\ x_{B31,B30} \end{bmatrix}
$$

Equation (2.36) has been written including the impact forces. But at the instant when there is no pounding, no impact elements are needed and they have to be deactivated in the solution procedure. To accomplish this, additional constraints must be introduced.

$$\begin{aligned} x_{11,B10} - x_{21,B20} - d_{11,21} \le 0 \;&\Rightarrow\; s_{11,21},\ c_{11,21} = 0 \\ x_{21,B20} - x_{31,B30} - d_{21,31} \le 0 \;&\Rightarrow\; s_{21,31},\ c_{21,31} = 0 \end{aligned} \tag{2.37}$$

We can also write the equation of motion for sliding of the bases for each of the three buildings. The equation for Building 1 will be

$$(m_{11} + M_{B11})\ddot{x}_{B11,B10} = -\mathrm{sgn}(\dot{x}_{B11,B10})F_{f_1} + F_{11} - (m_{11} + M_{B11})\ddot{x}_{B10} - m_{11}\ddot{x}_{11,B11} \tag{2.38}$$

for Building 2

$$(m_{21} + M_{B21})\ddot{x}_{B21,B20} = -\mathrm{sgn}(\dot{x}_{B21,B20})F_{f_2} + F_{21} - (m_{21} + M_{B21})\ddot{x}_{B20} - m_{21}\ddot{x}_{21,B21} \tag{2.39}$$

and for Building 3

$$(m_{31} + M_{B31})\ddot{x}_{B31,B30} = -\mathrm{sgn}(\dot{x}_{B31,B30})F_{f_3} + F_{31} - (m_{31} + M_{B31})\ddot{x}_{B30} - m_{31}\ddot{x}_{31,B31} \tag{2.40}$$

Since all the three buildings may not slide at the same time, thus some or all of the above equations may not be applied at a given time. For any of the buildings during no sliding, the sliding acceleration and velocity is zero for that building.

The conditions for sliding for each of the three buildings is similar to the one derived earlier in section 2.2.1. During sliding, following inequalities will hold. In particular, for Building 1

$$\left|(m_{11}+M_{B11})\ddot{x}_{B10}+m_{11}\ddot{x}_{B11,B10}+m_{11}\ddot{x}_{11,B11}-F_{11}\right|>\mu_1(m_{11}+M_{B11})g \tag{2.41}$$

and for Building 2

$$\left|(m_{21}+M_{B21})\ddot{x}_{B20}+m_{21}\ddot{x}_{B21,B20}+m_{21}\ddot{x}_{21,B21}-F_{21}\right|>\mu_2(m_{21}+M_{B21})g \tag{2.42}$$

and for Building 3

$$\left|(m_{31}+M_{B31})\ddot{x}_{B30}+m_{31}\ddot{x}_{B31,B30}+m_{31}\ddot{x}_{31,B31}-F_{31}\right|>\mu_3(m_{31}+M_{B31})g \tag{2.43}$$

2.3.2. TWO DEGREE OF FREEDOM SYSTEMS

In this section the dynamic equations for a two two-degree of freedom building model with both base isolation and pounding between the buildings is developed. The lumped mass model of these buildings is illustrated in figure 2.14. The coupled equations of motion for Building 1 are

$$\begin{aligned}&\begin{bmatrix} m_{11} & 0 \\ 0 & m_{12}\end{bmatrix}\begin{bmatrix}\ddot{x}_{11,B11}\\ \ddot{x}_{12,B11}\end{bmatrix}+\begin{bmatrix} c_{11}+c_{12} & -c_{12} \\ -c_{12} & c_{12}\end{bmatrix}\begin{bmatrix}\dot{x}_{11,B11}\\ \dot{x}_{12,B11}\end{bmatrix}+\begin{bmatrix} k_{11}+k_{12} & -k_{12} \\ -k_{12} & k_{12}\end{bmatrix}\begin{bmatrix}x_{11,B11}\\ x_{12,B11}\end{bmatrix}\\ &+\begin{bmatrix} s_{11,21}(x_{11,B10}-x_{21,B20}-d_{11,21})+c_{11,21}(\dot{x}_{11,B10}-\dot{x}_{21,B20}) \\ s_{12,22}(x_{12,B10}-x_{22,B20}-d_{12,22})+c_{12,22}(\dot{x}_{12,B10}-\dot{x}_{22,B20})\end{bmatrix}=\begin{bmatrix}F_{11}\\ F_{12}\end{bmatrix}\\ &-\begin{bmatrix} m_{11} & 0 \\ 0 & m_{12}\end{bmatrix}\begin{bmatrix}\ddot{x}_{B11,B10}\\ \ddot{x}_{B11,B10}\end{bmatrix}-\begin{bmatrix} m_{11} & 0 \\ 0 & m_{12}\end{bmatrix}\begin{bmatrix}\ddot{x}_{B10}\\ \ddot{x}_{B10}\end{bmatrix}\end{aligned} \tag{2.44}$$

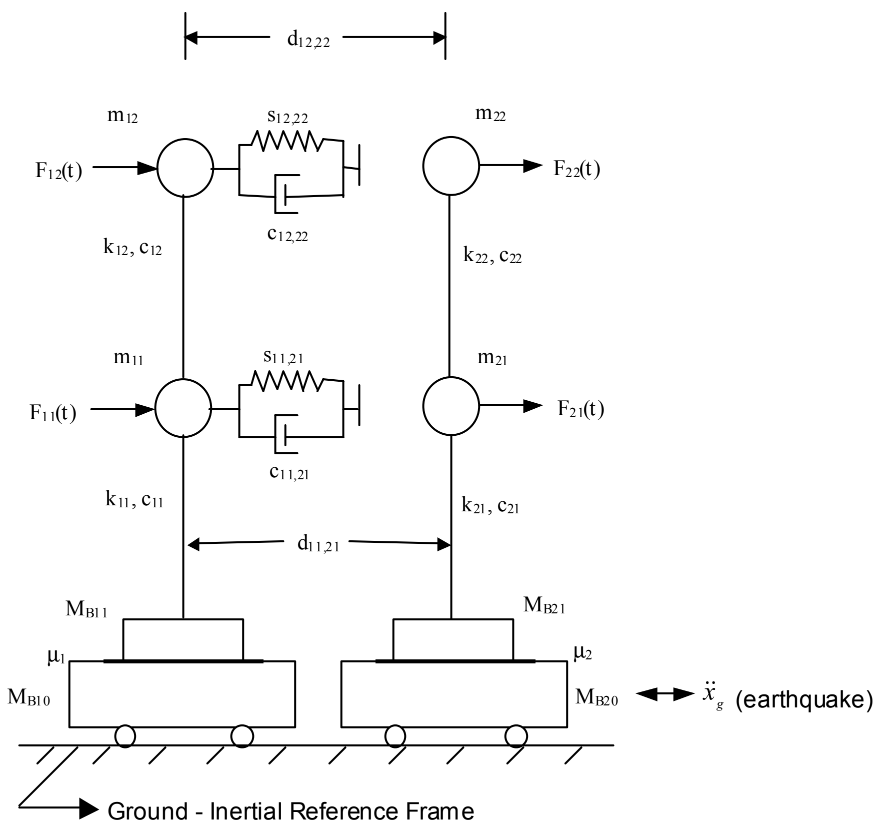

Fig 2.14 Schematic diagram of the two adjacent 2-DOF base isolated systems

and similarly for Building 2

$$\begin{bmatrix} m_{21} & 0 \\ 0 & m_{22} \end{bmatrix}\begin{bmatrix} \ddot{x}_{21,B21} \\ \ddot{x}_{22,B21} \end{bmatrix}+\begin{bmatrix} c_{21}+c_{22} & -c_{22} \\ -c_{22} & c_{22} \end{bmatrix}\begin{bmatrix} \dot{x}_{21,B21} \\ \dot{x}_{22,B21} \end{bmatrix}+\begin{bmatrix} k_{21}+k_{22} & -k_{22} \\ -k_{22} & k_{22} \end{bmatrix}\begin{bmatrix} x_{21,B21} \\ x_{22,B21} \end{bmatrix}$$
$$-\begin{bmatrix} s_{11,21}(x_{11,B10}-x_{21,B20}-d_{11,21})+c_{11,21}(\dot{x}_{11,B10}-\dot{x}_{21,B20}) \\ s_{12,22}(x_{12,B10}-x_{22,B20}-d_{12,22})+c_{12,22}(\dot{x}_{12,B10}-\dot{x}_{22,B20}) \end{bmatrix}=\begin{bmatrix} F_{21} \\ F_{22} \end{bmatrix} \quad (2.45)$$
$$-\begin{bmatrix} m_{21} & 0 \\ 0 & m_{22} \end{bmatrix}\begin{bmatrix} \ddot{x}_{B21,B20} \\ \ddot{x}_{B21,B20} \end{bmatrix}-\begin{bmatrix} m_{21} & 0 \\ 0 & m_{22} \end{bmatrix}\begin{bmatrix} \ddot{x}_{B20} \\ \ddot{x}_{B20} \end{bmatrix}$$

Combining equations (2.44) and (2.45) in to a single matrix equation yields

$$\begin{bmatrix} m_{11} & 0 & 0 & 0 \\ 0 & m_{12} & 0 & 0 \\ 0 & 0 & m_{21} & 0 \\ 0 & 0 & 0 & m_{22} \end{bmatrix}\begin{bmatrix} \ddot{x}_{11,B11} \\ \ddot{x}_{12,B11} \\ \ddot{x}_{21,B21} \\ \ddot{x}_{22,B21} \end{bmatrix}+\begin{bmatrix} c_{11}+c_{12} & -c_{12} & 0 & 0 \\ -c_{12} & c_{12} & 0 & 0 \\ 0 & 0 & c_{21}+c_{22} & -c_{22} \\ 0 & 0 & -c_{22} & c_{22} \end{bmatrix}\begin{bmatrix} \dot{x}_{11,B11} \\ \dot{x}_{12,B11} \\ \dot{x}_{21,B21} \\ \dot{x}_{22,B21} \end{bmatrix}+$$
$$\begin{bmatrix} k_{11}+k_{12} & -k_{12} & 0 & 0 \\ -k_{12} & k_{12} & 0 & 0 \\ 0 & 0 & k_{21}+k_{22} & -k_{22} \\ 0 & 0 & -k_{22} & k_{22} \end{bmatrix}\begin{bmatrix} x_{11,B11} \\ x_{12,B11} \\ x_{21,B21} \\ x_{22,B21} \end{bmatrix}+\begin{bmatrix} s_{11,21}(x_{11,B10}-x_{21,B20}-d_{11,21})+c_{11,21}(\dot{x}_{11,B10}-\dot{x}_{21,B20}) \\ s_{12,22}(x_{12,B10}-x_{22,B20}-d_{12,22})+c_{12,22}(\dot{x}_{12,B10}-\dot{x}_{22,B20}) \\ -s_{11,21}(x_{11,B10}-x_{21,B20}-d_{11,21})-c_{11,21}(\dot{x}_{11,B10}-\dot{x}_{21,B20}) \\ -s_{12,22}(x_{12,B10}-x_{22,B20}-d_{12,22})-c_{12,22}(\dot{x}_{12,B10}-\dot{x}_{22,B20}) \end{bmatrix}$$
$$=\begin{bmatrix} F_{11} \\ F_{12} \\ F_{21} \\ F_{22} \end{bmatrix}-\begin{bmatrix} m_{11} & 0 & 0 & 0 \\ 0 & m_{12} & 0 & 0 \\ 0 & 0 & m_{21} & 0 \\ 0 & 0 & 0 & m_{22} \end{bmatrix}\begin{bmatrix} \ddot{x}_{B11,B10} \\ \ddot{x}_{B11,B10} \\ \ddot{x}_{B21,B20} \\ \ddot{x}_{B21,B20} \end{bmatrix}-\begin{bmatrix} m_{11} & 0 & 0 & 0 \\ 0 & m_{12} & 0 & 0 \\ 0 & 0 & m_{21} & 0 \\ 0 & 0 & 0 & m_{22} \end{bmatrix}\begin{bmatrix} \ddot{x}_{B10} \\ \ddot{x}_{B10} \\ \ddot{x}_{B20} \\ \ddot{x}_{B20} \end{bmatrix}$$
(2.46)

Following the grouping of terms as before, one obtains

$$
\begin{aligned}
&\begin{bmatrix} m_{11} & 0 & 0 & 0 \\ 0 & m_{12} & 0 & 0 \\ 0 & 0 & m_{21} & 0 \\ 0 & 0 & 0 & m_{22} \end{bmatrix}
\begin{bmatrix} \ddot{x}_{11,B11} \\ \ddot{x}_{12,B11} \\ \ddot{x}_{21,B21} \\ \ddot{x}_{22,B21} \end{bmatrix}
+\left(\begin{bmatrix} c_{11}+c_{12} & -c_{12} & 0 & 0 \\ -c_{12} & c_{12} & 0 & 0 \\ 0 & 0 & c_{21}+c_{22} & -c_{22} \\ 0 & 0 & -c_{22} & c_{22} \end{bmatrix}
+\begin{bmatrix} c_{11,21} & 0 & -c_{11,21} & 0 \\ 0 & c_{12,22} & 0 & -c_{12,22} \\ -c_{11,21} & 0 & c_{11,21} & 0 \\ 0 & -c_{12,22} & 0 & c_{12,22} \end{bmatrix}\right)
\begin{bmatrix} \dot{x}_{11,B11} \\ \dot{x}_{12,B11} \\ \dot{x}_{21,B21} \\ \dot{x}_{22,B21} \end{bmatrix} \\
&+\left(\begin{bmatrix} k_{11}+k_{12} & -k_{12} & 0 & 0 \\ -k_{12} & k_{12} & 0 & 0 \\ 0 & 0 & k_{21}+k_{22} & -k_{22} \\ 0 & 0 & -k_{22} & k_{22} \end{bmatrix}
+\begin{bmatrix} s_{11,21} & 0 & -s_{11,21} & 0 \\ 0 & s_{12,22} & 0 & -s_{12,22} \\ -s_{11,21} & 0 & s_{11,21} & 0 \\ 0 & -s_{12,22} & 0 & s_{12,22} \end{bmatrix}\right)
\begin{bmatrix} x_{11,B11} \\ x_{12,B11} \\ x_{21,B21} \\ x_{22,B21} \end{bmatrix} \\
&+\begin{bmatrix} c_{11,21} & 0 & -c_{11,21} & 0 \\ 0 & c_{12,22} & 0 & -c_{12,22} \\ -c_{11,21} & 0 & c_{11,21} & 0 \\ 0 & -c_{12,22} & 0 & c_{12,22} \end{bmatrix}
\begin{bmatrix} \dot{x}_{B11,B10} \\ \dot{x}_{B11,B10} \\ \dot{x}_{B21,B20} \\ \dot{x}_{B21,B20} \end{bmatrix}
+\begin{bmatrix} s_{11,21} & 0 & -s_{11,21} & 0 \\ 0 & s_{12,22} & 0 & -s_{12,22} \\ -s_{11,21} & 0 & s_{11,21} & 0 \\ 0 & -s_{12,22} & 0 & s_{12,22} \end{bmatrix}
\begin{bmatrix} x_{B11,B10} \\ x_{B11,B10} \\ x_{B21,B20} \\ x_{B21,B20} \end{bmatrix} \\
&+\begin{bmatrix} -s_{11,21}d_{11,21} \\ -s_{12,22}d_{12,22} \\ s_{11,21}d_{11,21} \\ s_{12,22}d_{12,22} \end{bmatrix}
=\begin{bmatrix} F_{11} \\ F_{12} \\ F_{21} \\ F_{22} \end{bmatrix}
-\begin{bmatrix} m_{11} & 0 & 0 & 0 \\ 0 & m_{12} & 0 & 0 \\ 0 & 0 & m_{21} & 0 \\ 0 & 0 & 0 & m_{22} \end{bmatrix}
\begin{bmatrix} \ddot{x}_{B11,B10} \\ \ddot{x}_{B11,B10} \\ \ddot{x}_{B21,B20} \\ \ddot{x}_{B21,B20} \end{bmatrix}
-\begin{bmatrix} m_{11} & 0 & 0 & 0 \\ 0 & m_{12} & 0 & 0 \\ 0 & 0 & m_{21} & 0 \\ 0 & 0 & 0 & m_{22} \end{bmatrix}
\begin{bmatrix} \ddot{x}_{B10} \\ \ddot{x}_{B10} \\ \ddot{x}_{B20} \\ \ddot{x}_{B20} \end{bmatrix}
\end{aligned}
\tag{2.47}
$$

and are subject to the following constraints

$$
\begin{aligned}
&x_{12,B10} - x_{22,B20} - d_{12,22} \le 0 \;\;\Rightarrow\;\; s_{12,22}, c_{12,22} = 0 \\
&x_{11,B10} - x_{21,B20} - d_{11,21} \le 0 \;\;\Rightarrow\;\; s_{11,21}, c_{11,21} = 0
\end{aligned}
\tag{2.48}
$$

The dynamic equations for sliding of these structures are similar to equation (2.32) of an isolated building in section 2.2.2. Thus for Building 1

$$
\begin{aligned}
(m_{11}+m_{12}+M_{B11})\ddot{x}_{B11,B10} = -\mathrm{sgn}(\dot{x}_{B11,B10})F_{f1} + F_{11} + F_{12} - (m_{11}+m_{12}+M_{B11})\ddot{x}_{B10} \\
- m_{11}\ddot{x}_{11,B11} - m_{12}\ddot{x}_{12,B11}
\end{aligned}
\tag{2.49}
$$

and for Building 2

$$(m_{21}+m_{22}+M_{B21})\ddot{x}_{B21,B20}=-\mathrm{sgn}(\dot{x}_{B21,B20})F_{f2}+F_{21}+F_{22}-(m_{21}+m_{22}+M_{B21})\ddot{x}_{B20} \\ -m_{21}\ddot{x}_{21,B21}-m_{22}\ddot{x}_{22,B21} \tag{2.50}$$

Furthermore, the conditions for sliding of Building 1 is

$$\left|(m_{11}+m_{12}+M_{B11})\ddot{x}_{B10}+(m_{11}+m_{12})\ddot{x}_{B11,B10}+m_{11}\ddot{x}_{11,B11}+m_{12}\ddot{x}_{12,B11}-F_{11}-F_{12}\right| \\ >\mu_1(m_{11}+m_{12}+M_{B11})g \tag{2.51}$$

and for Building 2 is

$$\left|(m_{21}+m_{22}+M_{B21})\ddot{x}_{B20}+(m_{21}+m_{22})\ddot{x}_{B21,B20}+m_{21}\ddot{x}_{21,B21}+m_{22}\ddot{x}_{22,B21}-F_{21}-F_{22}\right| \\ >\mu_2(m_{21}+m_{22}+M_{B21})g \tag{2.52}$$

The same argument holds for these inequalities as for other base isolated models presented previously. The models developed in section 2.3.1 and 2.3.2 are the most general for single degree and two-degree of freedom systems respectively. By using appropriate values of coefficient of friction in the sliding isolation system and the separation between the buildings, we can obtain the models given in other sections. Moreover it is now possible to investigate the pounding between base isolated and fixed base buildings that are adjacent to each other. Two separate computer programs have been written in MATLAB to solve the system of buildings given in section 2.3.1 and section 2.3.2. These programs have been included in this report and given in Appendix 3 and 4 respectively. Since the models developed in section 2.3 are the most general, these computer programs can be used to solve the models developed in other sections.

3. VALIDATION OF THE NUMERICAL MODEL: FIXED BASE

The dynamic equations derived in sections 2.3.1 and 2.3.2 are the most general and can be used to analyze other types of adjacent building systems by changing the spacing between the buildings or by varying the friction coefficient in the base isolation. These dynamic equations are second order differential equations and can be solved numerically by converting them into a first order differential equation and then solved using the ordinary differential equation solvers provided by MATLAB. The first order differential equation form of these dynamic equations have been given in Appendix 1 and 2 for single degree and two degree of freedom systems respectively. The MATLAB files developed for this study are also included and given in Appendix 3 and 4 respectively.

The chapter has been divided into two sections. In the first section, the response of a single fixed base two degree of freedom building subjected to harmonic base excitation is obtained analytically using the modal analysis method. The MATLAB code used for modal analysis is given in Appendix 5. This semi-analytic solution is used to check the correctness of the response calculated subsequently using numerical technique for which the MATLAB program is given in Appendix 4. In the second section we will consider pounding between the fixed base buildings subjected to earthquake ground motion and analyze the response of this system.

3.1. SEMI-ANALYTIC SOLUTION: 2-DOF BUILDING

Modal analysis method is used to obtain the solution. Modal analysis is used for the solution of dynamic equation of multi degree of freedom system. In modal analysis as described in detail in Clough and Penzien (1993), coupled equations of a multi degree of freedom system are uncoupled using the normal coordinate transformation in equation (3.1).

$$\{x\} = [\varphi]\{q\} \tag{3.1}$$

Here, $\{x\}$ is the geometric coordinate vector and $\{q\}$ is the generalized coordinate vector. The matrix $[\varphi]$ is the mode shape matrix. This transformation can be used to convert the N coupled equations in (3.2) to N uncoupled equations given by (3.3).

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = \{p(t)\} \tag{3.2}$$

$$\ddot{q}_n + 2\xi_n\omega_n\dot{q}_n + \omega_n{}^2 q_n = \frac{P_n(t)}{M_n}, \qquad n = 1, 2, ..., N \tag{3.3}$$

Where, $M_n = \varphi_n{}^T m\varphi_n$ and $P_n(t) = \varphi_n{}^T p(t)$ are generalized mass and load respectively for the nth mode and ξ_n and ω_n are the critical damping ratio and natural frequency in that mode. Once generalized coordinates have been obtained from equation (3.3), they can be used to calculate the response of the system using equation (3.1). The MATLAB files to obtain the modal analysis solution are given in Appendix 5.

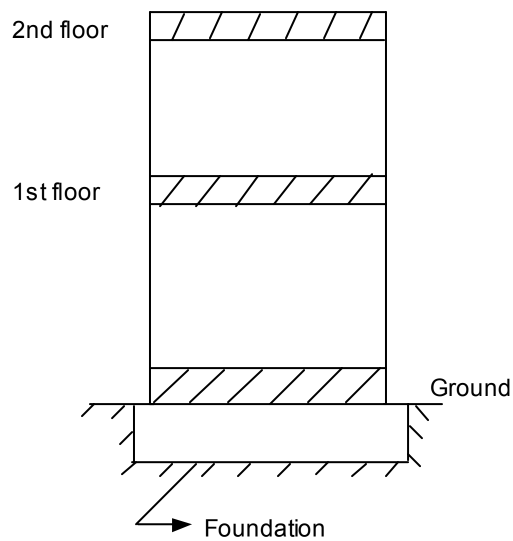

Fig 3.1 Reference building used for analysis

Consider the building shown in figure 3.1. This shear building has been adopted as a reference building and is used throughout the text. The data for the building have been adopted from an example given in the book by Paz (1985). The mass and the stiffness matrix for the lumped mass model of the building are specified as

$$M = \begin{bmatrix} m_{11} & 0 \\ 0 & m_{12} \end{bmatrix} = \begin{bmatrix} 26268 & 0 \\ 0 & 17512 \end{bmatrix} Kgs \tag{3.4}$$

$$K = \begin{bmatrix} k_{11} + k_{12} & -k_{12} \\ -k_{12} & k_{12} \end{bmatrix} = \begin{bmatrix} 42028800 & -17512000 \\ -17512000 & 17512000 \end{bmatrix} N/m \tag{3.5}$$

The mode shapes and natural frequencies of the system obtained using the above mass and stiffness matrices are

$$\varphi = \begin{bmatrix} 0.4951 & 0.7601 \\ 0.8688 & -0.6498 \end{bmatrix} \tag{3.6}$$

$$\omega = \begin{bmatrix} 20.74 \\ 46.58 \end{bmatrix} rad/\sec \tag{3.7}$$

$$T = \begin{bmatrix} 0.303 \\ 0.135 \end{bmatrix} \sec \tag{3.8}$$

The damping matrix is obtained by assuming that it is proportional to the stiffness matrix (Clough and Penzien 1993). This is assumed to be a good approximation for developing the damping matrix since our model has only two modes. Moreover it is simpler to find the dashpot constants for the building using this damping. One can write

$$C = a_1 K \tag{3.9}$$

This can be simplified to

$$\xi_n = \frac{a_1 \omega_n}{2} \tag{3.10}$$

By assuming 1% damping in the first mode one finds that $a_1 = 0.000964$ and this corresponds to a damping ratio of 2.24% in the second mode. The resulting damping matrix of the system from (3.9), then is

$$C = \begin{bmatrix} 40515.76 & -16881.57 \\ -16881.57 & 16881.57 \end{bmatrix} N.\text{sec}/\,m \tag{3.11}$$

This building has been used as the benchmark for all the analysis in this research study. For the semi-analytic solution, the building is subjected to a sinusoidal earthquake ground motion. This analysis is done for two different frequencies of the ground motion. One can write

$$\ddot{x}_g = \ddot{x}_B = A \sin \Omega t$$

$$\text{Where } A = 10 \text{ m/sec}^2,\ \Omega = 2 \text{ and } 10 \text{ rad/sec}^2 \tag{3.12}$$

Figure 3.2 presents the response of this two-degree of freedom system.

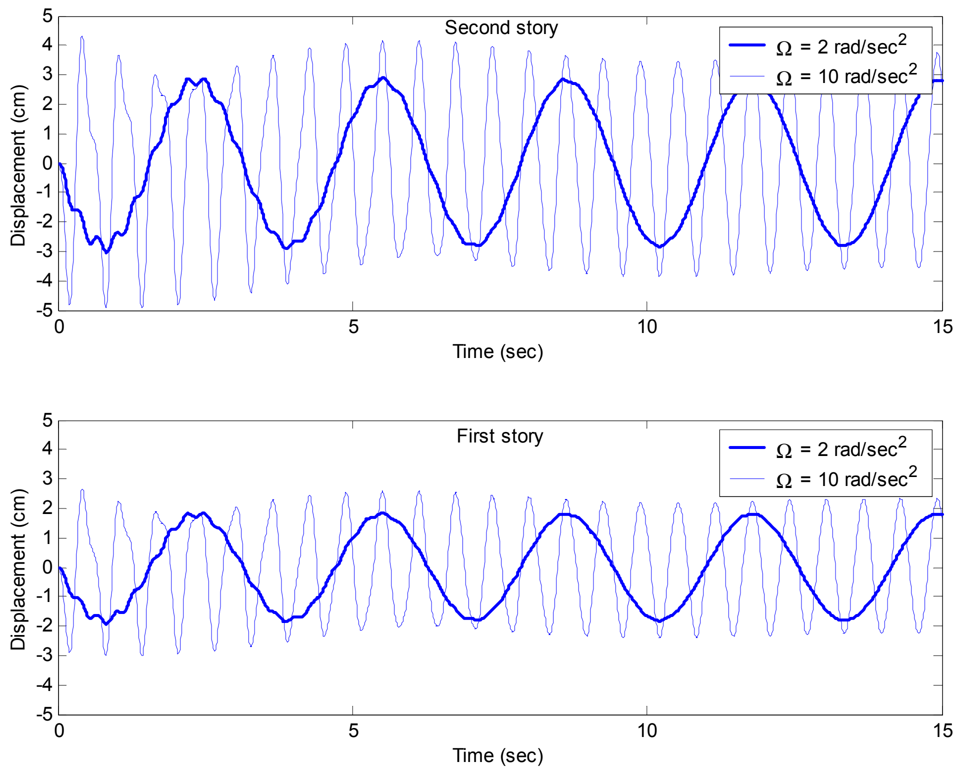

Fig 3.2 Semi-analytical solution for 2-DOF system. Second floor response; First floor response

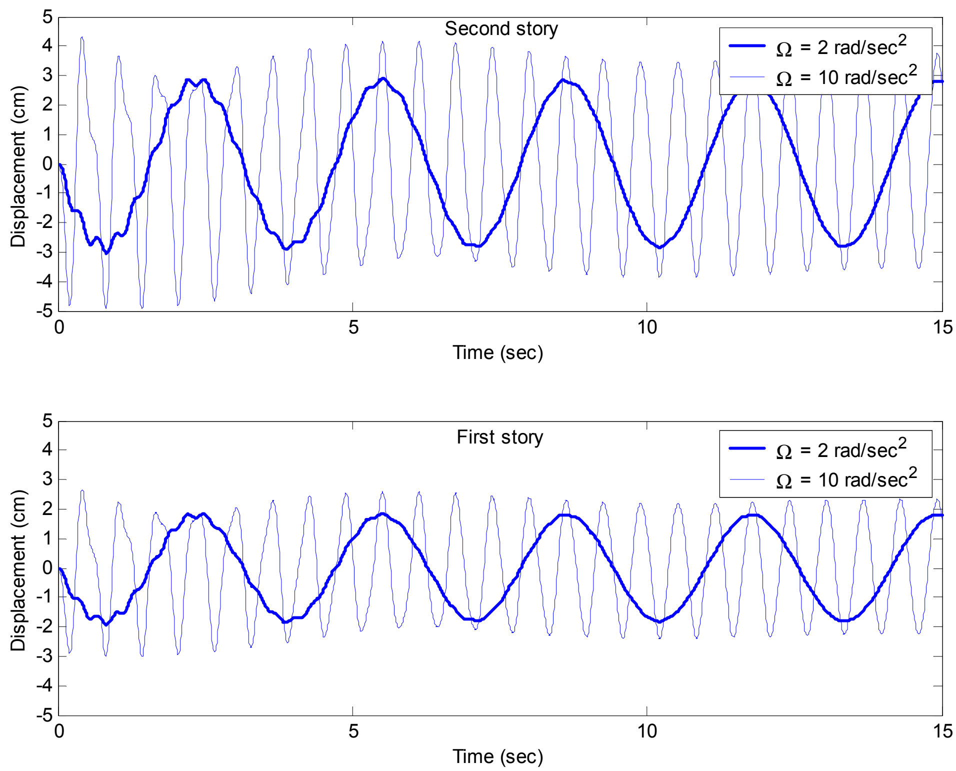

Fig 3.3 Numerical solution for 2-DOF system. Second floor response; First floor response

Numerical solution of the isolated two degree of freedom system subjected to harmonic ground motion was obtained using the MATLAB file given in Appendix 4 and is presented in figure 3.3. This response is nearly identical to the semi-analytic solution one get from modal analysis. Thus the MATLAB program in Appendix 4 is deemed accurate and will be used in this study. This check however does not confirm the accuracy of the program when pounding or sliding takes place. The systems in which pounding or sliding takes place can't be solved analytically and have to be solved numerically since these are non-linear behaviors.

3.2. POUNDING IN FIXED BASE BUILDINGS: 2-DOF MODEL

In this research investigation, the focus of the numerical simulation is two degree of freedom system. Three adjacent building configurations were investigated in the process of studying building pounding. The first configuration is the common one in which both the adjacent buildings are fixed base. The second configuration investigates the pounding between a base isolated and a fixed base building and in the final configuration, both of the adjacent buildings are base isolated. These configurations have been illustrated in table 3.1. It also illustrates the section numbers in which these configurations have been dealt. In this section, response will be calculated for adjacent buildings whose dynamic equation has been derived in section 2.1.2. The buildings were modeled as two degree of freedom systems and dynamic equations derived assuming pounding can take place at any of the floor levels. Buildings have been subjected to earthquake ground motion with time history of ground acceleration of El Centro earthquake. Other earthquakes used in this study include those from Mexico City, Loma Prieta and the Northridge earthquake.

The time history of earthquakes and statistics used to describe them are presented in figure 3.4 and table 3.2 respectively. It should be noted that the earthquakes have been arranged in the increasing order of their peak ground acceleration.

Table 3.1 Adjacent building configurations used in this study

Building 1	Building 2	Section number
Fixed	Fixed: Stiff Fixed: Flexible	3.2
Base isolated	Fixed: Stiff Fixed: Flexible	4.2
Base isolated	Base isolated: Stiff Base isolated: Flexible	4.3

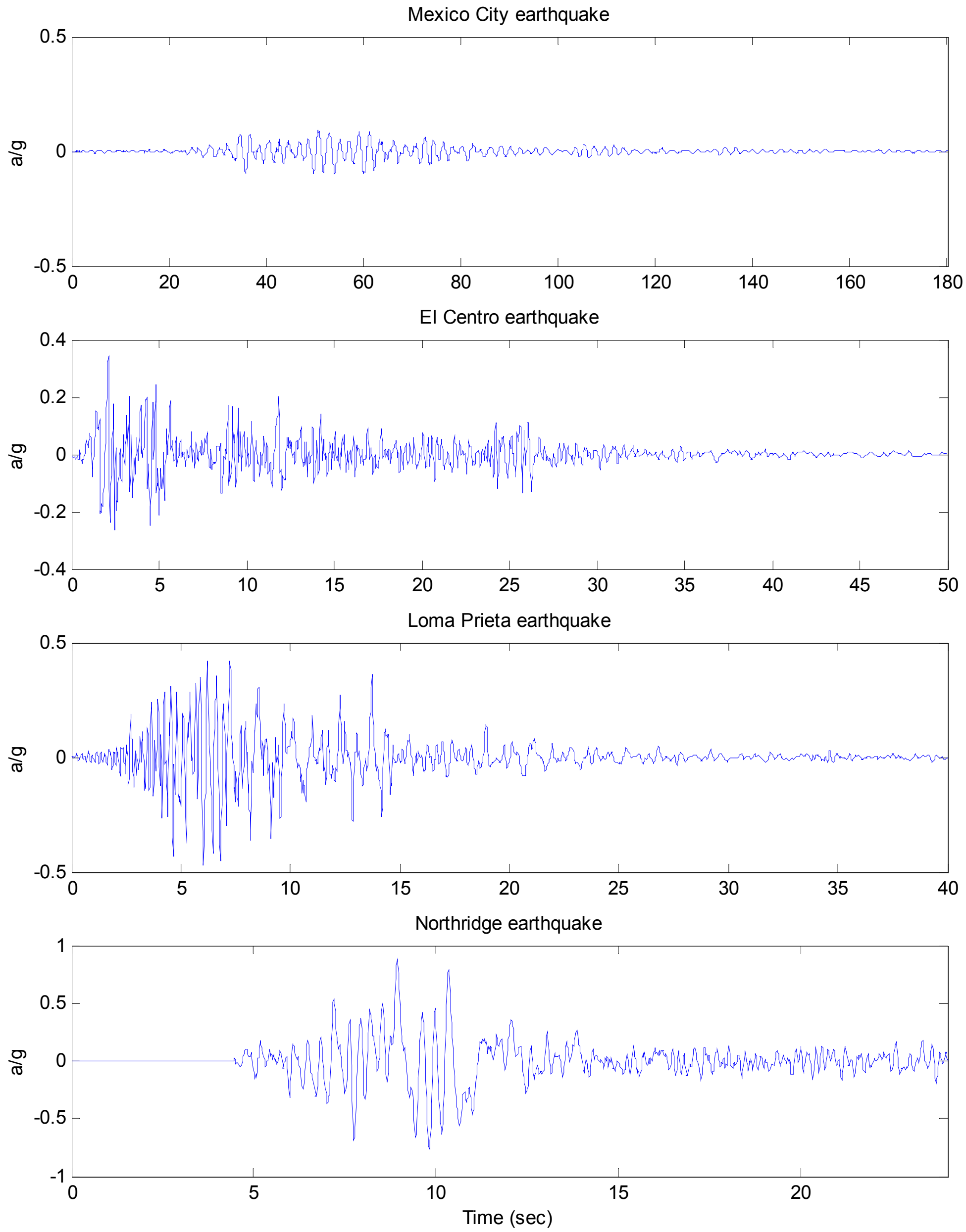

Fig 3.4 Acceleration time histories for the earthquakes

Table 3.2 Characterization of environment – Statistics for earthquakes

Earthquake	Max Accl.	Mean (m/sec^2)	Std deviation (m/sec^2)	Skewness	CoE
1985 Mexico City S00E component	0.100 g	-0.000080674	0.2135	-0.3805	4.7465
1940 El Centro S00E component	0.340 g	0.00059944	0.4679	0.4476	8.0111
1989 Loma Prieta	0.470 g	-0.00038051	0.8266	-0.1113	7.3894
1994 Northridge, taken from pier 10 on the I5 14 ramp C	0.874 g	0.0009669	1.6829	0.0462	5.9197

The difference in duration and amplitude are quite evident in figure 3.4. The most evident statistical characteristic is the coefficient of excess. This clearly shows the non-gaussian nature of these records.

Adjacent buildings shown in figure 3.5 have been used for analysis. Building 1 is the reference building described in section 3.1 and Building 2 has a varying stiffness with respect to the reference building. A stiffer or a more flexible Building 2 is considered which have mass half or double that of the reference building. Time periods and frequencies of Building 2 have been depicted in table 3.3. The damping of 1% in the first mode is kept same as that for the reference building by reconstructing the damping matrix as its mass is varied. Gap between the buildings have also been varied.

Impact stiffness has been calculated as done by Anagnostopoulos *et al* 1992 where in the article it was stated that "the stiffnesses of the impact springs were assigned values such that the local periods of the mass-impact springs were below the lowest translational periods of the pounding buildings". The dashpot constant of the impact element is calculated using these formulas (Anagnostopoulos 1988).

$$c = 2\xi_i \sqrt{\frac{k m_1 m_2}{m_1 + m_2}} \quad \text{and} \quad \xi_i = \frac{-\ln r}{\sqrt{\pi^2 + (\ln r)^2}} \tag{3.13}$$

Here, k is the stiffness of the impact element and ξ_i is the damping ratio and depends on the coefficient of restitution between the colliding masses for the inelastic impact, r is the coefficient of restitution whose value is 1 for completely elastic impact and 0 for completely plastic impact. Since pounding of buildings is neither fully elastic nor fully plastic, we can assume its value to be 0.50 as done by Anagnostopoulos *et al* 1992.

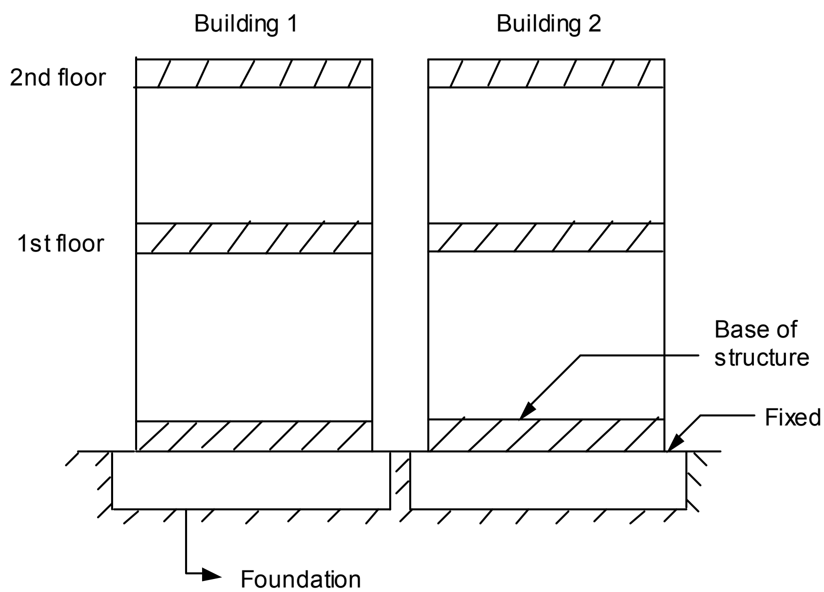

Fig 3.5 Adjacent fixed base buildings

Table 3.3 Properties of Building 2

Properties	Building 2	
	Stiff	Flexible
Mass with respect to reference building	Half	Twice
First mode time period (sec)	0.2142	0.4284
Second mode time period (sec)	0.0954	0.1908
First mode frequency (rad/sec)	29.33	14.66
Second mode frequency (rad/sec)	65.87	32.94

In this section the response of the buildings studied in section 2.1.2 have been investigated for El Centro earthquake. It was found from response time history that the impact between the lower floors is very less. Hence all the analysis and any reference to displacement response of building in this study refer to top floor responses.

Figure 3.6 shows response time history of Building 1 and also the impact forces during pounding for the case when buildings are spaced 2 cm apart and Building 2 is flexible. Asterisk on the time history response indicates pounding at that time. Figure 3.7 shows the variation in number of impacts with increase in gap between the buildings for different stiffness of Building 2. It can be seen from this graph that number of total impacts decreases with increase in gap and it decreases faster in case when Building 2 is stiff. Also the total number of impacts is higher when Building 2 is flexible as compared to the case when it is stiff. In both of these cases the number of in-phase is lower than out of phase impacts. In-phase impacts are those that occur while the buildings are moving in the same direction and out of phase impacts are those that occur while they move in the opposite direction and towards each other. It is expected that out of phase impacts decreases the response of the pounding buildings but can cause more local damage whereas in-phase impacts increases the response but cause less local damage.

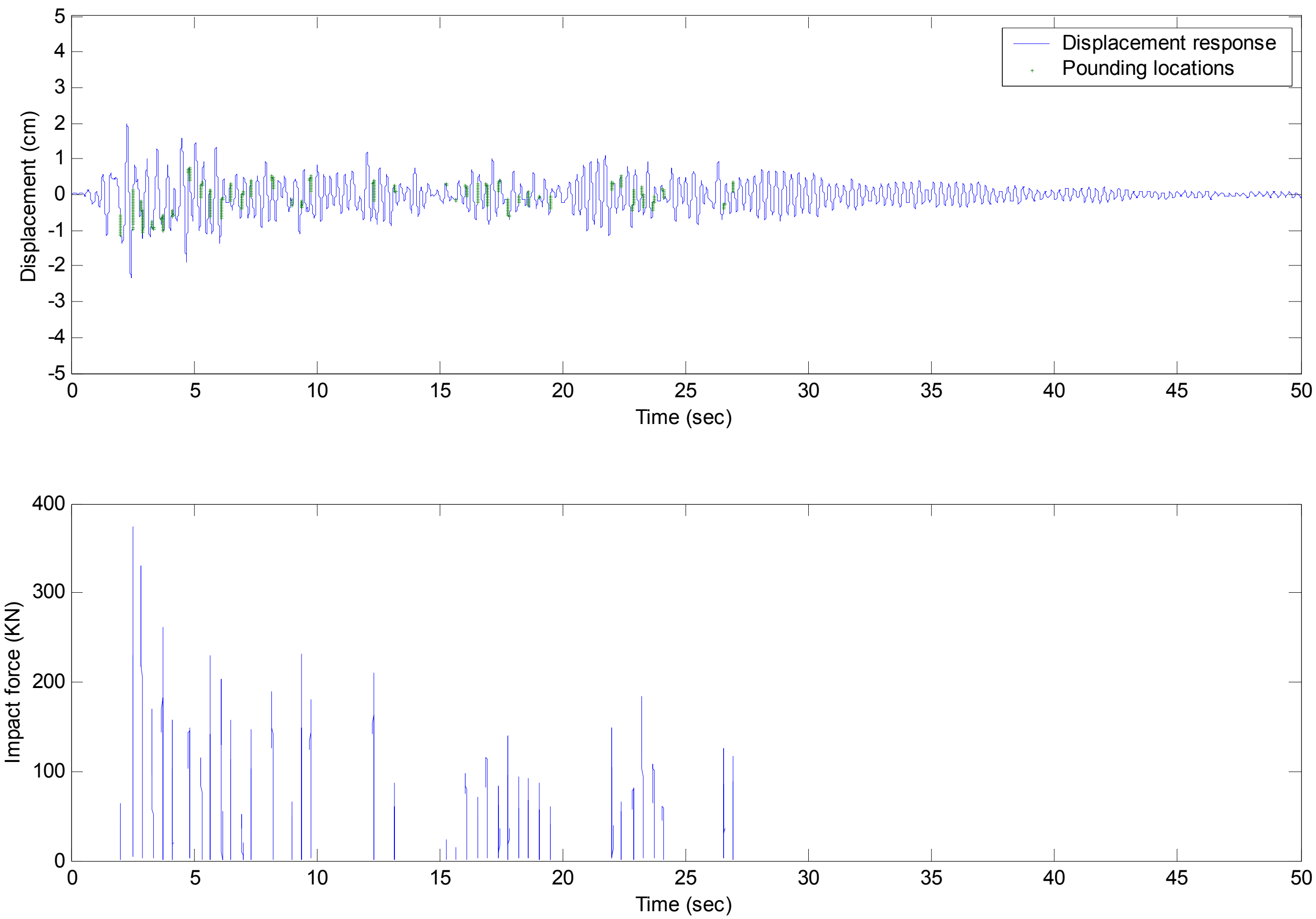

Fig 3.6 Response time history of Building 1; Impact forces; Building 2: Flexible, Gap = 2 cm

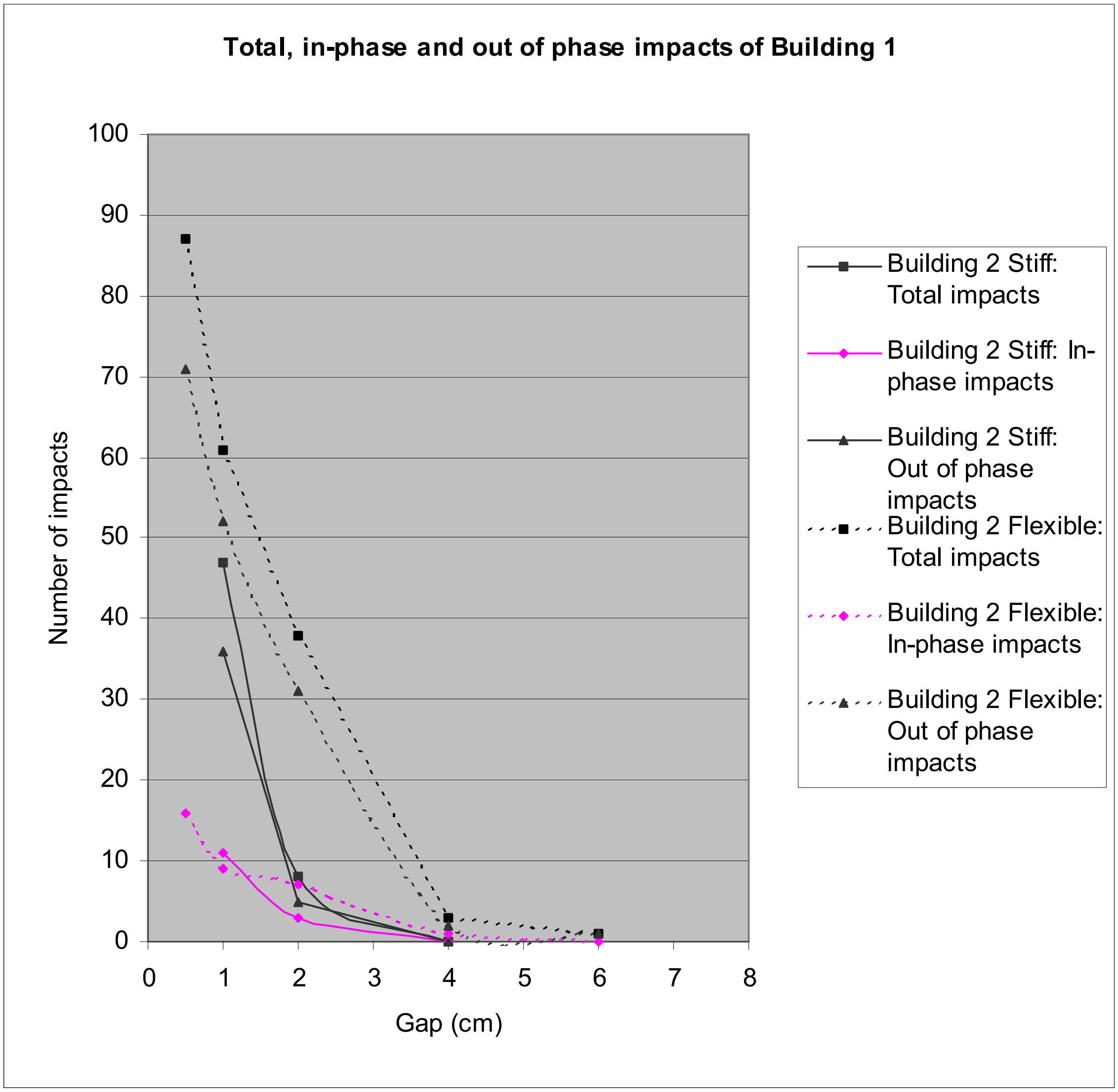

Fig 3.7 Variation in number of impacts with increasing gap between the buildings

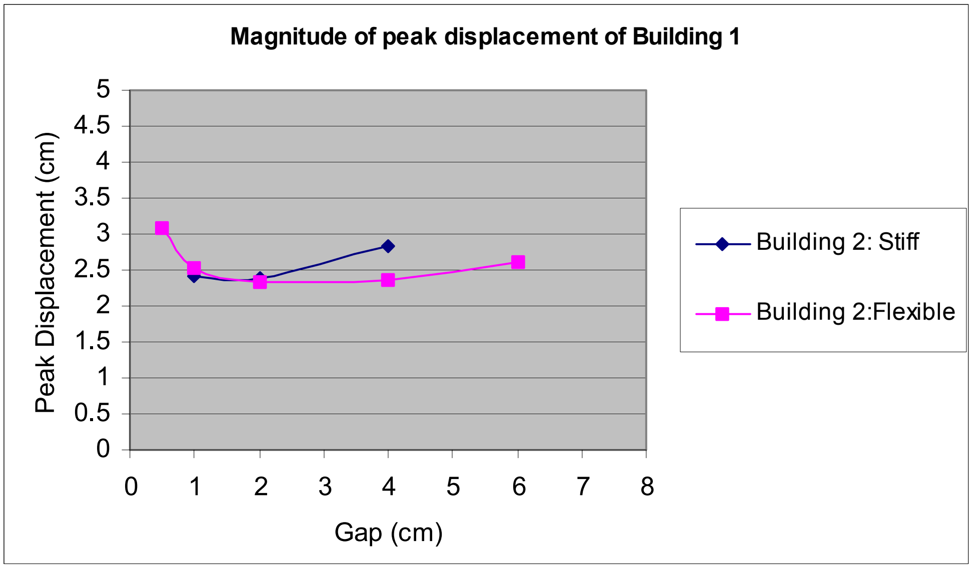

Fig 3.8 Variation of peak displacement with increasing gap between the buildings

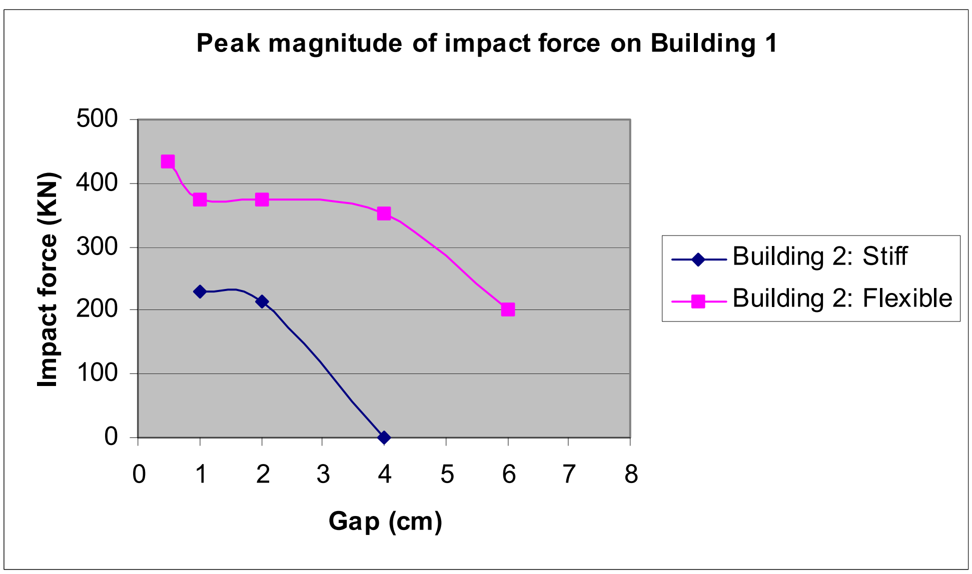

Fig 3.9 Variation in peak impact force with increasing gap between the buildings

Figure 3.8 and 3.9 shows the variation in magnitude of peak displacement and peak impact force as the gap between the buildings is varied. It was observed that peak displacement decreased with increase in pounding when the gap is reduced, probably due to more out of phase impacts. Also peak displacement was found to increase as the gap size is reduced to a very small distance (although the overall response was found to be less), probably due to very high amount of pounding in which there can be lot of in-phase impacts. This peak displacement was higher when Building 2 is flexible. This is in compliance with the study done by Anagnostopoulos 1988, who reached the same conclusion for fixed base buildings with very small gap. The maximum impact force was found to be higher in flexible building case than the stiff building case and in both the cases it reduced with increase in the gap.

4. SIMULATION OF BUIILDING POUNDING WITH BASE ISOLATION

The numerical results presented in this chapter were obtained using the MATLAB program given in Appendix 4. This chapter is divided into three sections. The first section deals with calculating the response of single base isolated building for a selected range of friction coefficient values. Four different earthquakes that cover a range of magnitudes are used in the building response simulation. In the second section the response of two adjacent buildings, one of which is base isolated and the other one having fixed base, is simulated. We will assume that Building 1 is base isolated and is the reference building. In the third section, response of two adjacent buildings, both of them being base isolated, is obtained.

4.1. SINGLE BASE ISOLATED BUILDING

In this section the response behavior of single base isolated two degree of freedom building is investigated. The reference building described earlier is now used and this provides a basis for later simulations. Here, mass of the building base is assumed to be half the mass of the lumped first floor. The response has been calculated for many coefficients of friction to examine the variation in response behavior. Figure 4.1 and 4.2 shows the variation of skewness and coefficient of excess for the total displacement response of second floor for the base isolated building subjected to different earthquakes. From these figures it can be seen that skewness and coefficient of excess for Mexico City earthquake is constant, since there is no sliding of the building at any time. Further, the skewness is very small and coefficient of excess is high due to low intensity of the earthquake. For the other earthquakes it can be seen that skewness first decreases and then increases as the friction coefficient increases. Coefficient of excess for these earthquakes tends towards one with an increase in friction coefficient.

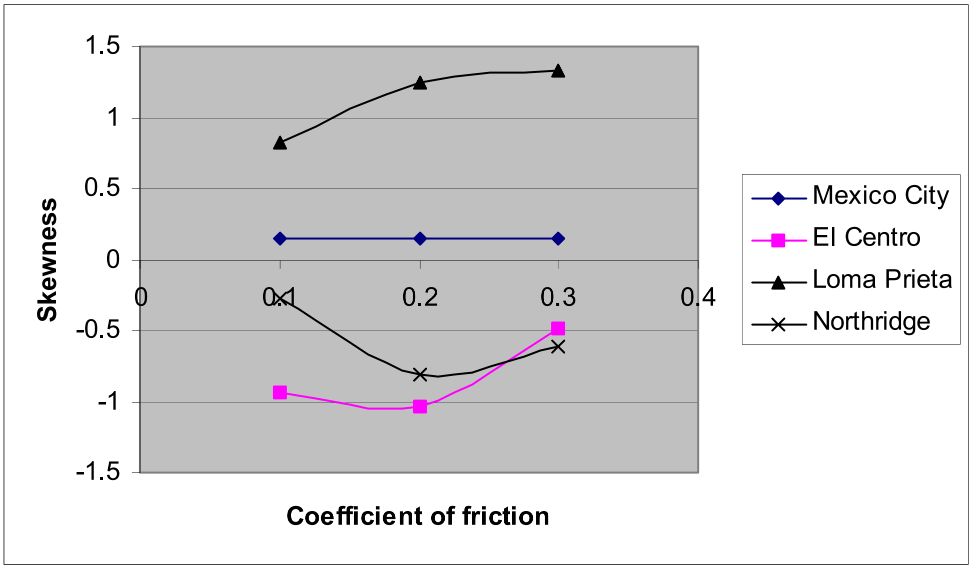

Fig 4.1 Skewness of total displacement response for different earthquakes

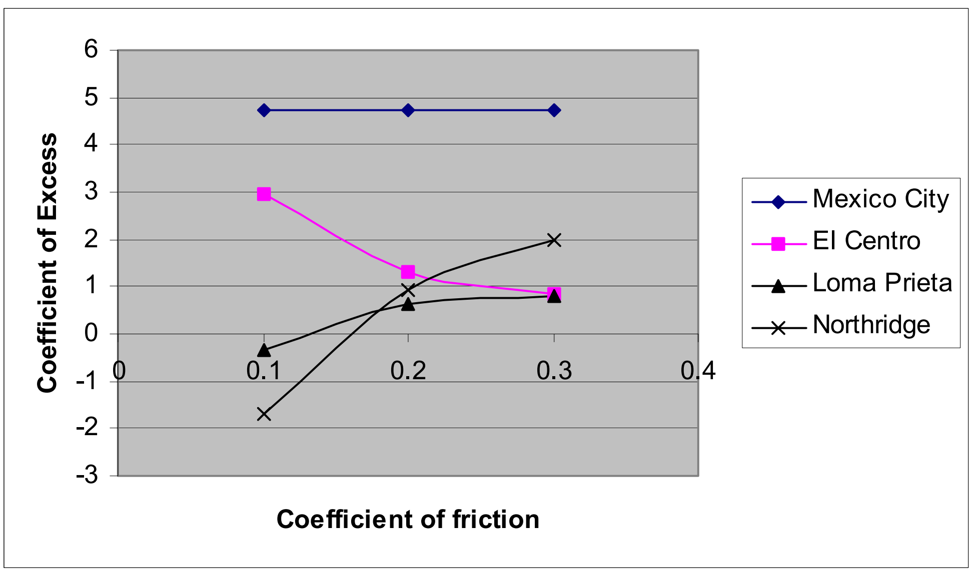

Fig 4.2 Coefficient of excess of total displacement response for different earthquakes

Variation of maximum relative displacement, maximum sliding displacement and maximum total displacement with coefficient of friction for different earthquakes have been given in figures 4.3, 4.4 and 4.5 respectively. With regards to the building sliding response, the Northridge case is the most dramatic as the response decrease significantly with increase in the friction coefficient. Since sliding displacement response is usually much higher than the relative displacement response, the variation in total displacement response is approximately same as that of sliding displacement for each of the earthquakes. One is surprised to note that sliding displacement have increased with increase in friction coefficient for Loma Prieta earthquake, which otherwise will be expected to decrease. The relative displacement response is lower for lower coefficients of friction as expected, which illustrates the advantage of base isolation of buildings. Since there is an upper limit on the shear force that can be transferred by the sliding interface to the superstructure, it can be seen that the relative displacement responses are approximately same for the three earthquakes of different intensities. Thus a base isolation system proves to be most useful for higher intensity earthquakes.

A series of histograms superimposed with a corresponding normal distribution curve for identical buildings with a specified coefficient of friction in the base isolation system are presented in figure 4.6. Intrestingly it is observed that Loma Prieta is the only earthquake for which the mean is on the negative side. It is not obvious as to why this occurs. Finally it was intresting to observe that only Mexico City earthquake resulted in a building response whose mean was close to zero.

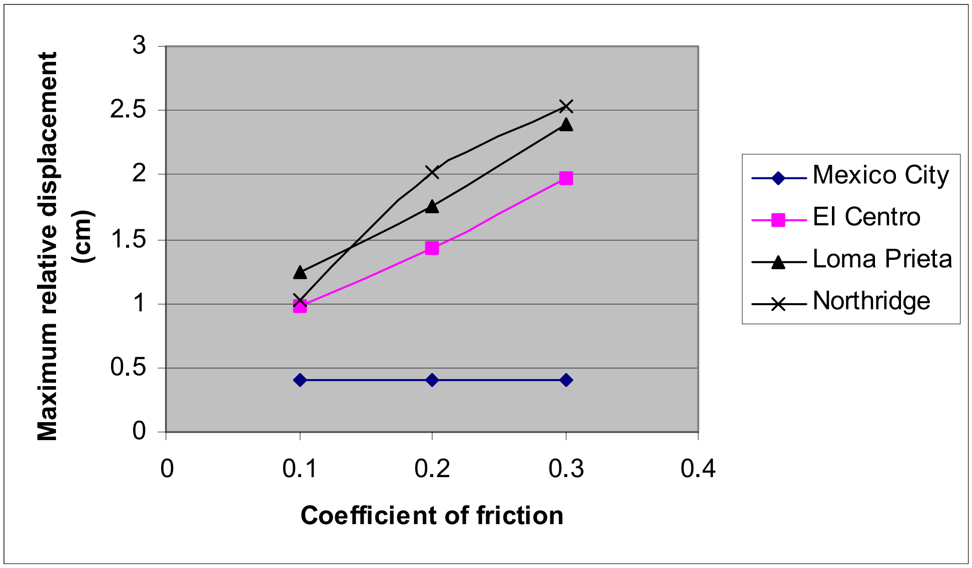

Fig 4.3 Maximum relative displacement response for different earthquakes

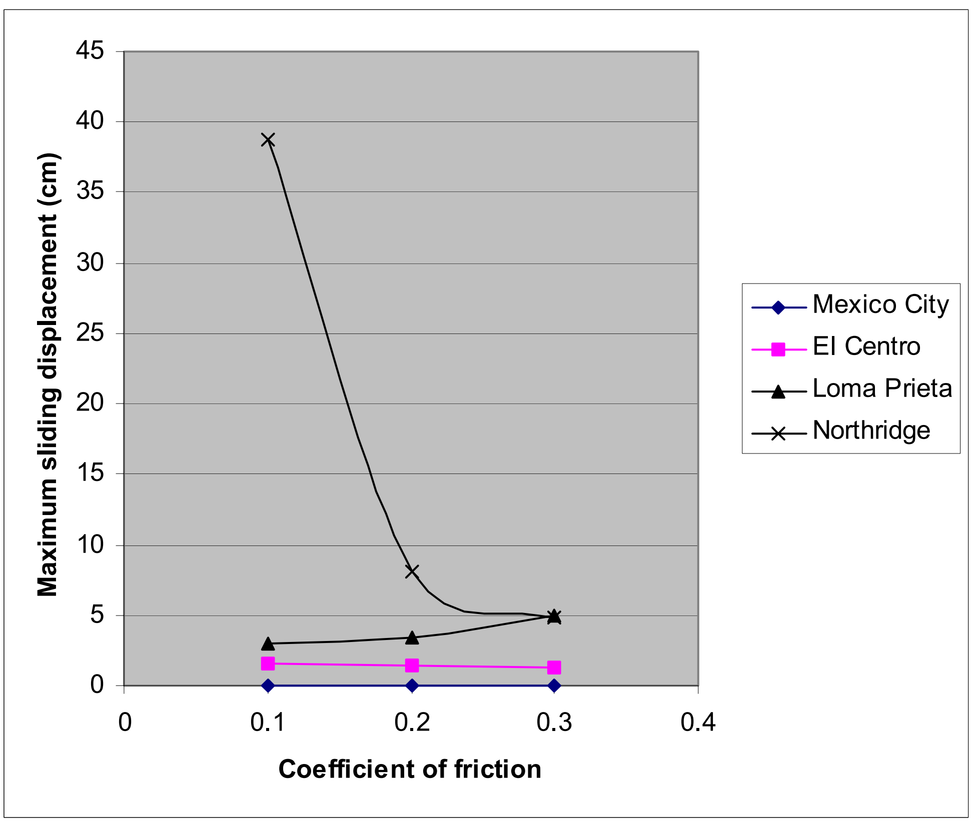

Fig 4.4 Maximum sliding displacement response for different earthquakes

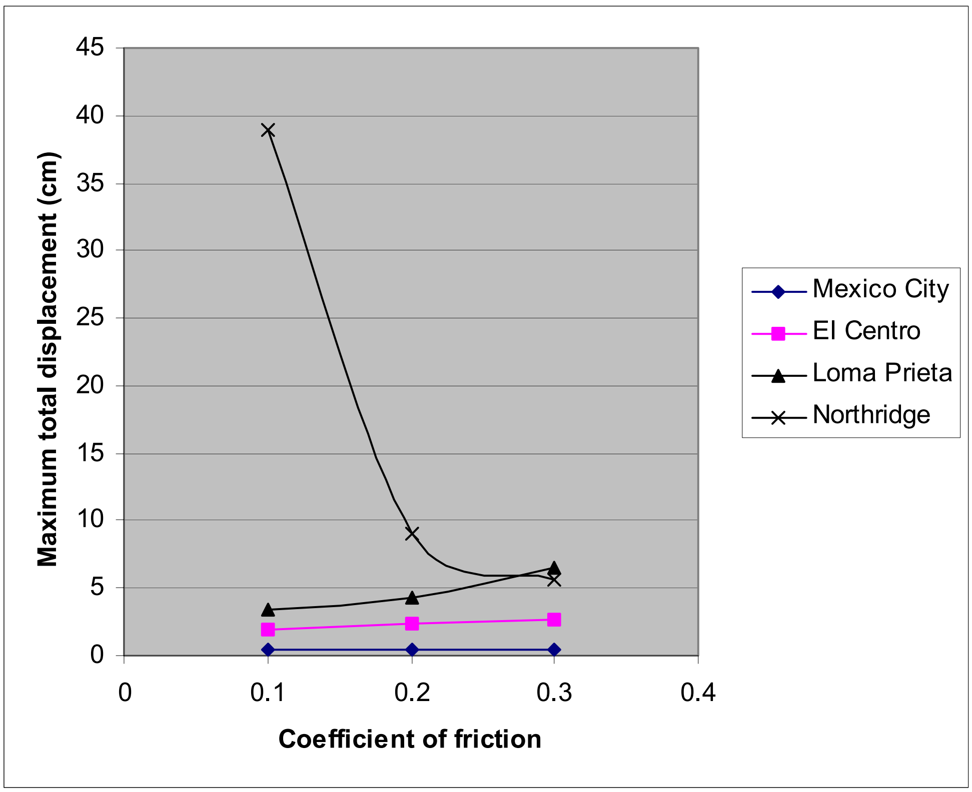

Fig 4.5 Maximum total displacement response for different earthquakes

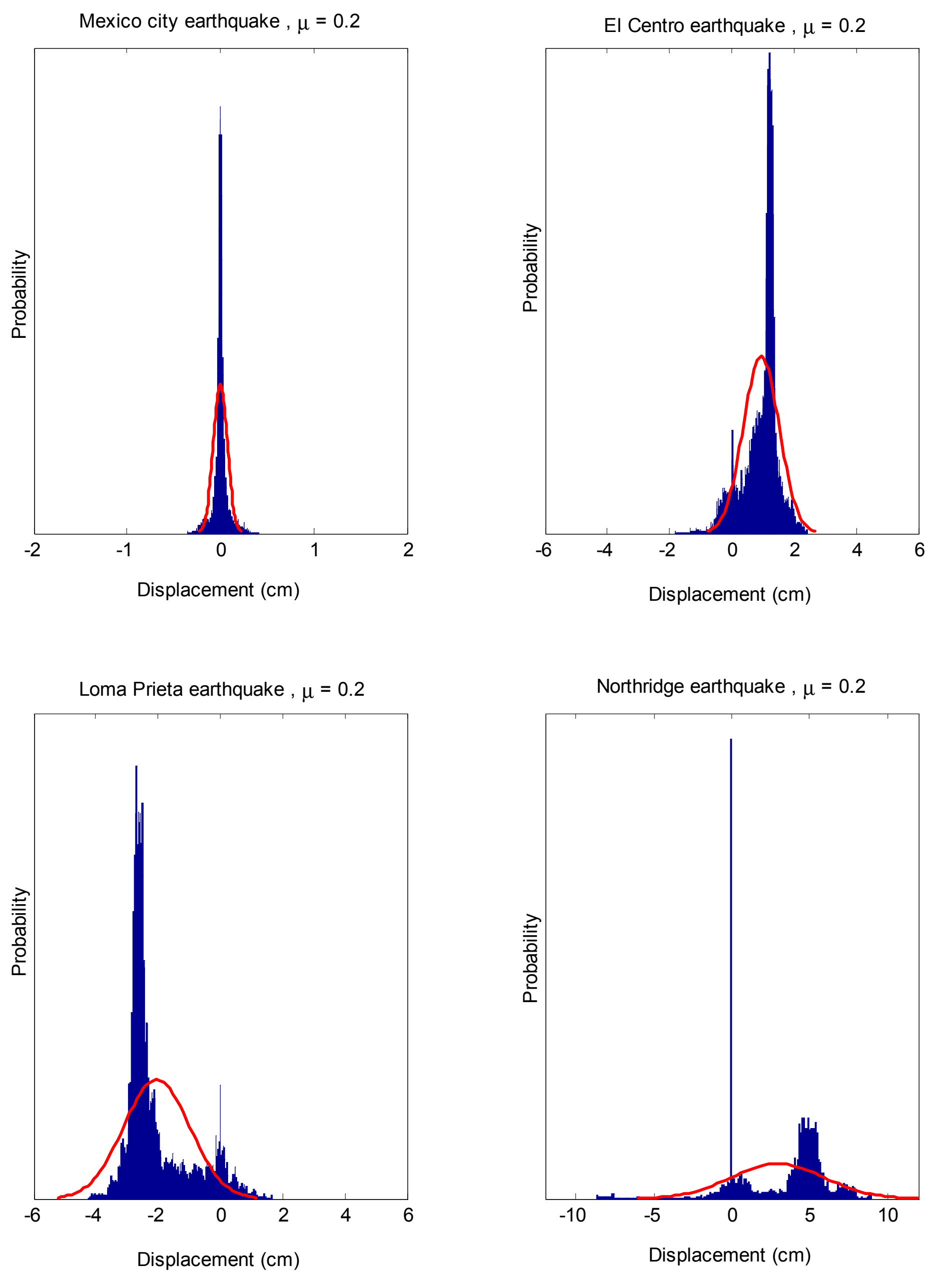

Fig 4.6 Histograms and normal probability curve for the displacement response of base isolated reference building subjected to different earthquakes

4.2. FIXED BASE AND BASE ISOLATED ADJACENT BUILDINGS

This section considers the response of a base isolated building that is closely placed to a fixed base building. The buildings are illustrated in figure 4.7 and it is assumed that Building 1 is the reference building and is base isolated. For the response simulations, Building 2 is either stiffer or more flexible than the reference building. In this study, the building system has been subjected to ground acceleration corresponding to the four selected earthquakes. Although the friction coefficient in base isolation will practically range from 0.05 to 0.2, simulations have been done and results presented for higher values also in order to see the response behavior.

The total number of impacts for different stiffness of Building 2 and for different friction coefficient in the sliding system of Building 1 is presented in figure 4.8. It can be seen that the total numbers of impacts were higher for stiffer version of Building 2 when coefficient of friction is low whereas they were higher for more flexible version of Building 2 when coefficient of friction is high. This occured since during El Centro earthquake, when friction coefficient is low, Building 1 was observed to slide towards Building 2 thus decreasing the gap between the buildings. Note that as the gap reduces, the impacts for stiffer system increase faster.

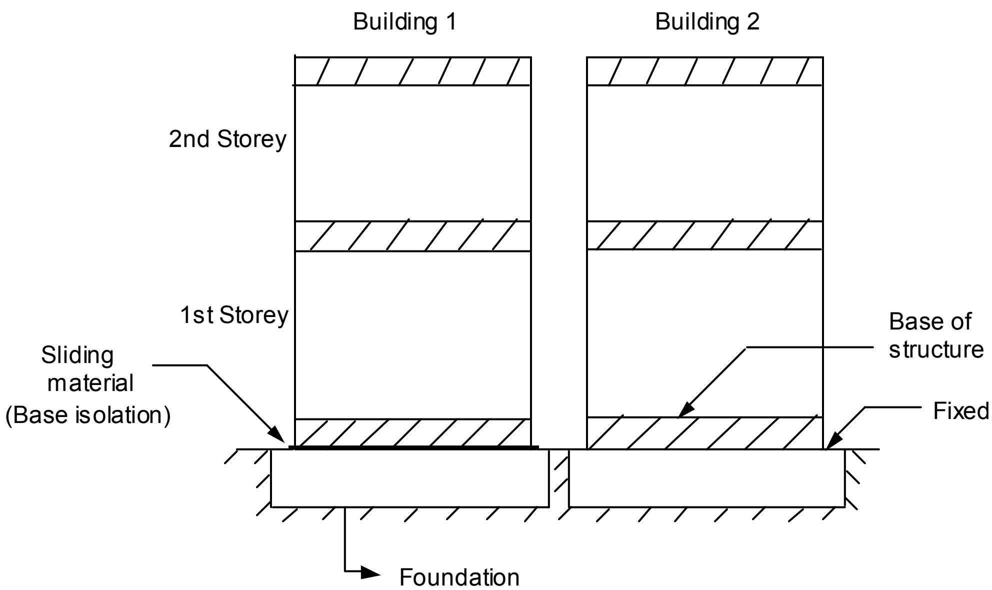

Fig 4.7 Adjacent base isolated and fixed base buildings

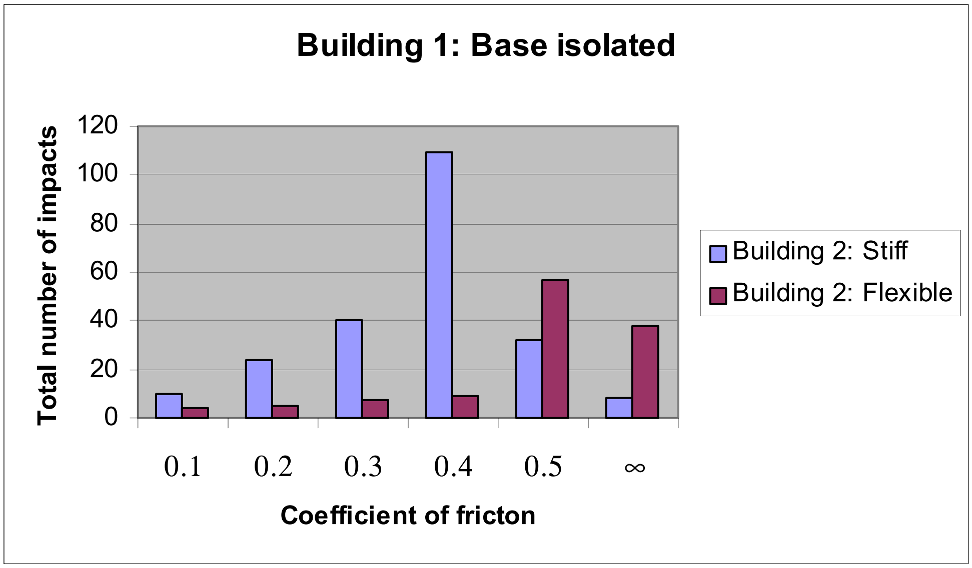

Fig 4.8 Total number of impacts as a function of Building 2 stiffness and base isolation friction coefficient in Building 1

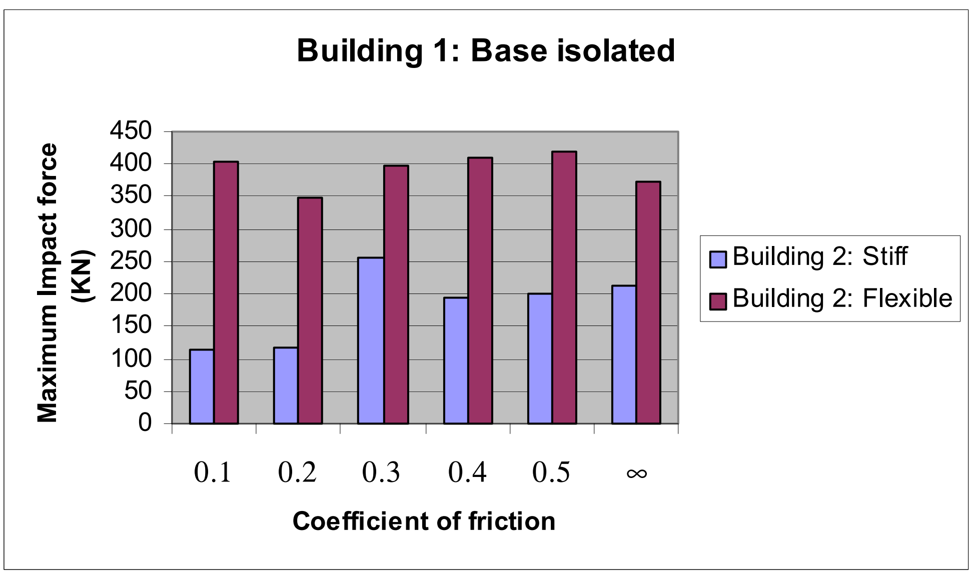

Fig 4.9 Maximum impact force as a function of Building 2 stiffness and base isolation friction coefficient in Building 1

Figure 4.9 shows the variation of maximum impact force and figure 4.10 presents the maximum duration of impacts. It was found that the impact force and duration of impacts was higher when Building 2 was flexible. For coefficient of friction equal to 0.3 and 0.4 when Building 2 is stiff, it was observed that there are some impacts with relatively high durations. From response time histories it was observed that in these cases Building 1 slid very close to Building 2. Due to decrease in gap, they acted as if sticking to each other and more so when friction coefficient was 0.4. Due to this, the impact duration was very high and it has not been shown in figure 4.10. This did not happen when Building 2 was flexible, since high magnitude of impact caused sliding and shifted Building 1 away from it, again increasing the gap between them. Figure 4.11 presents information on the nature of the maximum magnitude of total displacement for second floor of Building 1. Note that the total displacement is measured with respect to the original location of the building. It was observed from the time histories of response that when Building 2 is flexible, the impact force was very high that caused the sliding of Building 1 in opposite direction resulting in a high sliding displacement. Whereas when Building 2 is stiff, lower impact forces do not cause much sliding. A higher total displacement in the mid-value range of friction coefficient was due to higher relative displacement that occurred for higher friction coefficient in addition to sliding displacement.

Table 4.1 presents some of the statistics for total displacement for varying friction coefficients in the base isolation system of Building 1 and stiff Building 2 with a gap between them equal to 2 centimeters. Table 4.2 shows the same statistics for different earthquakes and friction coefficient equal to 0.2. In case of Loma Prieta and Northridge earthquakes, very high and impractical sliding displacements due to pounding were observed at a gap of 2 centimeters. Thus the gap between the buildings, when subjected to these earthquakes, was increased to 4 and 8 centimeters respectively.

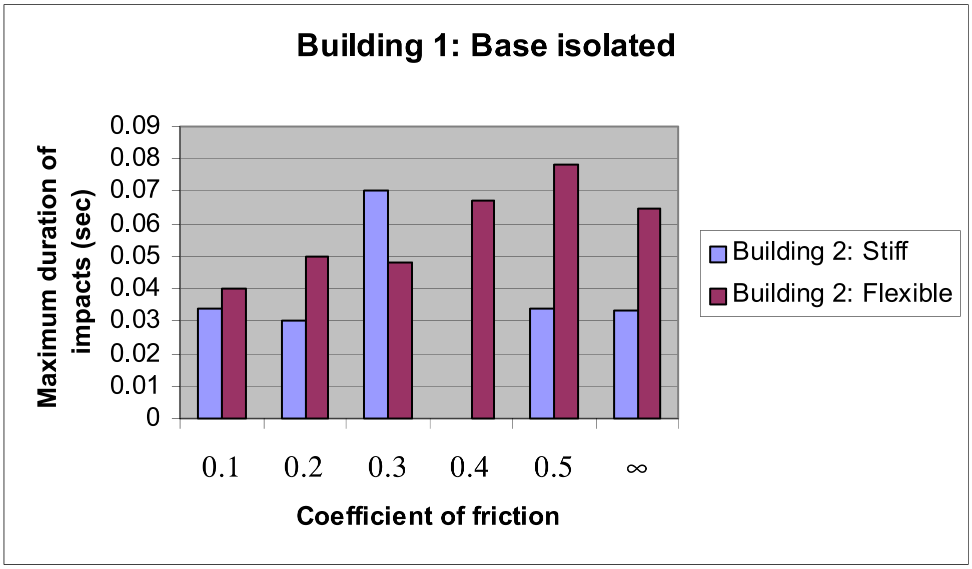

Fig 4.10 Maximum duration of impacts as a function of Building 2 stiffness and base isolation friction coefficient in Building 1

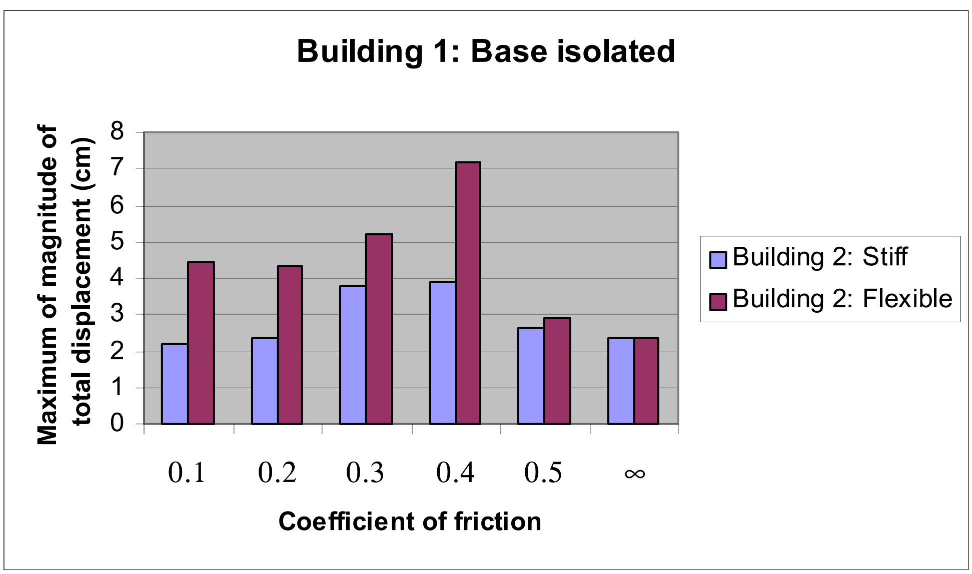

Fig 4.11 Total displacement as a function of Building 2 stiffness and base isolation friction coefficient in Building 1

Table 4.1 Response behavior characterization for varying friction coefficients. Building 1: base isolated, Building 2: stiff

Adjacent buildings with stiff Building 2							
Statistics	Single reference building, fixed base	Friction coefficients for base isolated Building 1					
		$\mu = \infty$	$\mu = 0.5$	$\mu = 0.4$	$\mu = 0.3$	$\mu = 0.2$	$\mu = 0.1$
Mean (cm)	0	-0.0021	0.77	2.11	-0.0067	0.71	0.36
Variance (cm)	0.49	0.4356	0.3364	0.8464	0.7921	0.3025	0.1936
Std deviation (cm)	0.7	0.66	0.58	0.92	0.89	0.55	0.44
Skewness	0.0246	-0.0107	-0.5361	-1.4938	0.8739	-0.3464	-0.4364
CoE	1.8417	0.9556	1.229	2.2515	1.5333	0.9919	2.3687
Total number of impacts	NA	8	32	109	40	24	10

Table 4.2 Response behavior characterization for earthquakes of increasing intensity. Building 1: base isolated, Building 2: stiff

Friction coefficient μ = 0.2 , Gap = 2 cm						
Earthquake	Mean (cm)	Std deviation (cm)	Skewness	CoE	Inphase impacts	Out of phase impacts
Mexico City	0	0.07486	0.1465	4.7465	0	0
El Centro	0.71	0.55	-0.3464	0.9919	6	18
Loma Prieta, Gap = 4 cm	-2.05	1.06	1.2448	0.6232	0	0
Northridge, Gap = 8 cm	2.36	2.58	-0.9539	2.4053	0	4

Table 4.3 Response behavior characterization for varying friction coefficients. Building 1: base isolated, Building 2: flexible

Adjacent buildings with flexible Building 2							
Statistics	Single reference building, fixed base	Friction coefficients for base isolated Building 1					
		$\mu = \infty$	$\mu = 0.5$	$\mu = 0.4$	$\mu = 0.3$	$\mu = 0.2$	$\mu = 0.1$
Mean (cm)	0	-0.027	0.5	-3.18	-0.172	-1.43	-1.45
Variance (cm)	0.49	0.16	0.2809	1.6384	0.5625	0.3844	0.2601
Std deviation (cm)	0.7	0.4	0.53	1.28	0.75	0.62	0.51
Skewness	0.0246	-0.1577	-1.111	1.6341	0.7415	0.192	1.0417
CoE	1.8417	2.8934	5.6526	3.2429	2.9527	1.9298	5.7851
Total number of impacts	NA	38	57	9	7	5	4

Table 4.4 Response behavior characterization for earthquakes of increasing intensity. Building 1: base isolated, Building 2: flexible

Friction coefficient μ = 0.2 , Gap = 2 cm						
Earthquake	Mean (cm)	Std deviation (cm)	Skewness	CoE	Inphase impacts	Out of phase impacts
Mexico City	0	0.07486	0.1465	4.7465	0	0
El Centro	-1.43	0.62	0.192	1.9298	2	3
Loma Prieta, Gap = 4 cm	-6.51	3.13	1.448	0.6515	4	4
Northridge, Gap = 8 cm	-9.53	7.8	0.3587	-1.7383	2	3

Table 4.3 and 4.4 are the corresponding tables when Building 2 is flexible. Comparing the mean of response in tables 4.2 and 4.4, it can be seen that the means shifts to left side or the negative side when Building 2 is flexible. This happens since the impact force for flexible building is large which causes large sliding displacements of Building 1. A similar phenomenon can be observed in table 4.1 and 4.3 where the buildings are subjected to El Centro earthquake and result have been presented for various values of coefficient of friction. It can also be observed in tables 4.2 and 4.4 that when the mean is positive, skewness is negative and vice versa. This happens due to peak in the distribution of response at the original position of the building that is at zero.

4.3. BASE ISOLATION IN BOTH ADJACENT BUILDINGS

The response behavior of two adjacent base isolated buildings is presented in this section. The system of buildings is same as shown in figure 4.7 except that here Building 2 is base isolated as well. It is assumed that the friction coefficient in the base isolation system of both the buildings is the same. As done in the last section, here also we will draw bar graphs for total number of impacts, maximum impact force, maximum duration of impacts and maximum of the magnitude of total displacements. These graphs have been given in figures 4.12, 4.13, 4.14 and 4.15 respectively. As previously shown, here also the total number of impacts is higher for stiffer Building 2 when the friction coefficient of sliding system is small and is higher for more flexible Building 2 when friction coefficient is large. An exception to this was seen for coefficient of friction equal to 0.1. In this case it was found that since friction coefficient is small, both the buildings slides and keep the distance between them approximately same. Thus there is very less pounding for a stiff Building 2 case. Maximum impact force and the maximum duration of impacts, as seen earlier, are generally higher when Building 2 is flexible. The maximum of the magnitude of total displacement is observed to reduce considerably for flexible Building 2 as seen in figure 4.15 as compared to figure 4.11 where only Building 1 was base isolated.

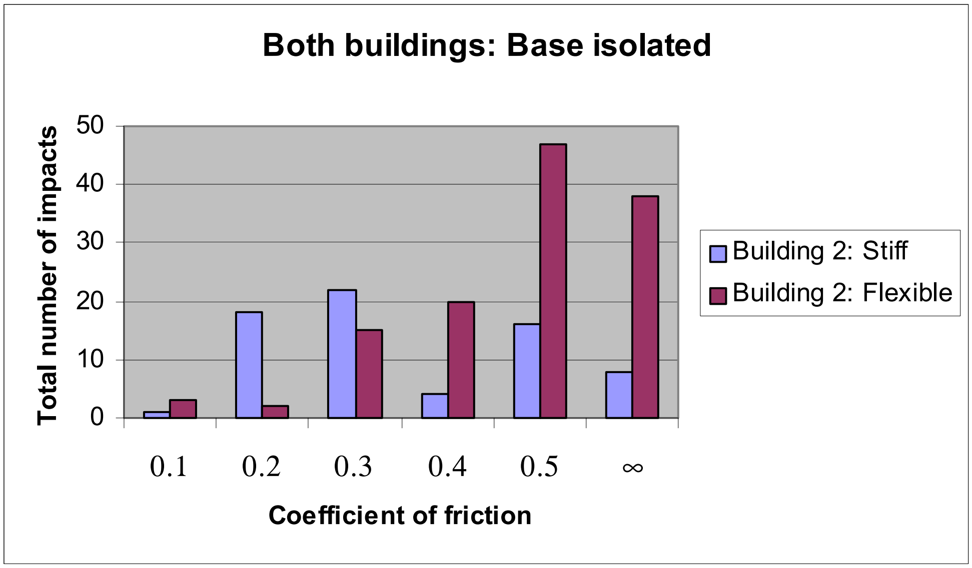

Fig 4.12 Total number of impacts as a function of Building 2 stiffness and base isolation friction coefficient

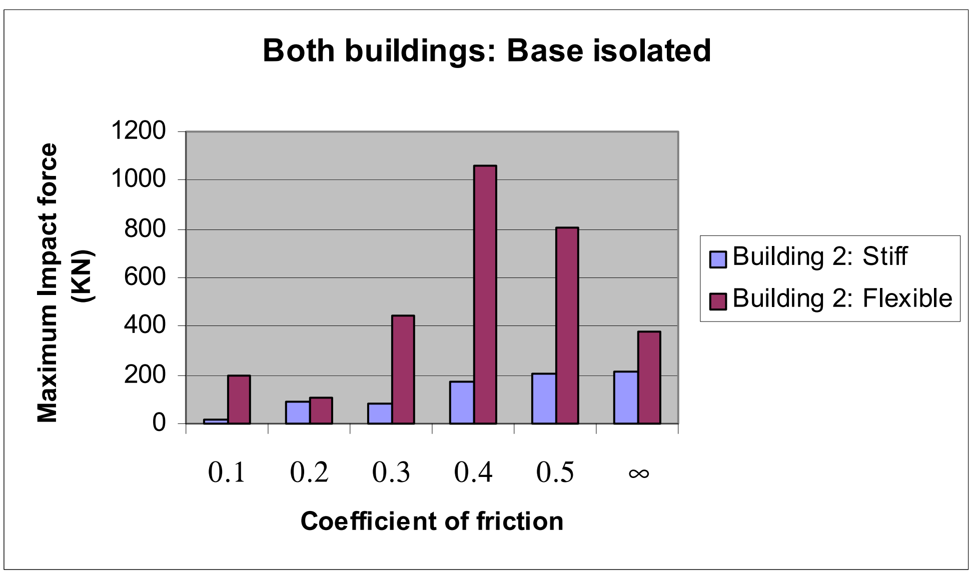

Fig 4.13 Maximum impact force as a function of Building 2 stiffness and base isolation friction coefficient

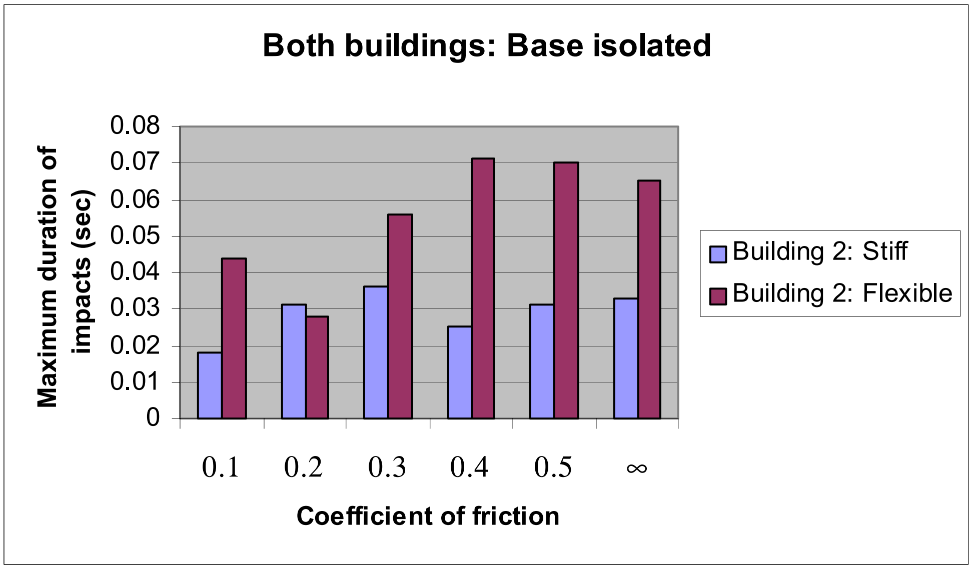

Fig 4.14 Maximum duration of impacts as a function of Building 2 stiffness and base isolation friction coefficient

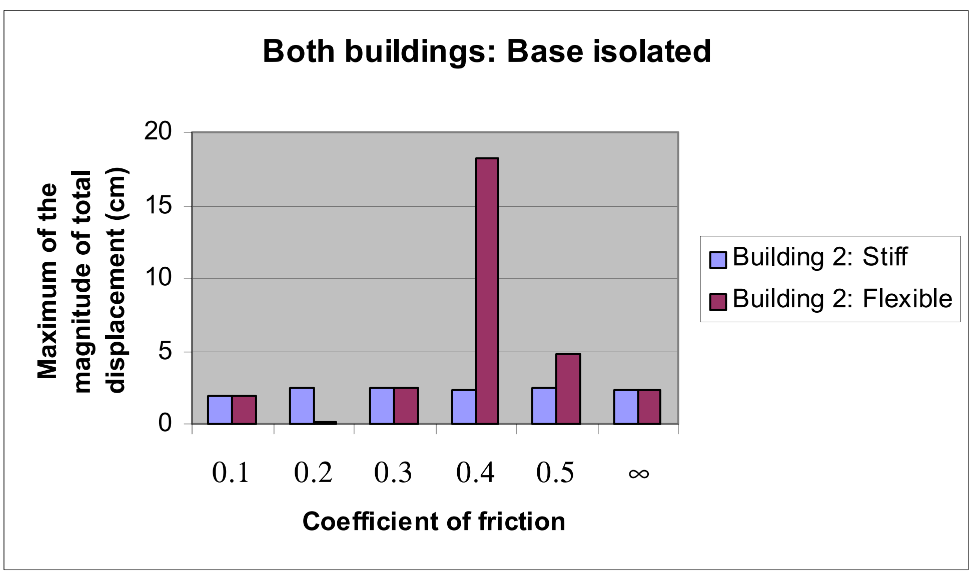

Fig 4.15 Maximum total displacement as a function of Building 2 stiffness and base isolation friction coefficient

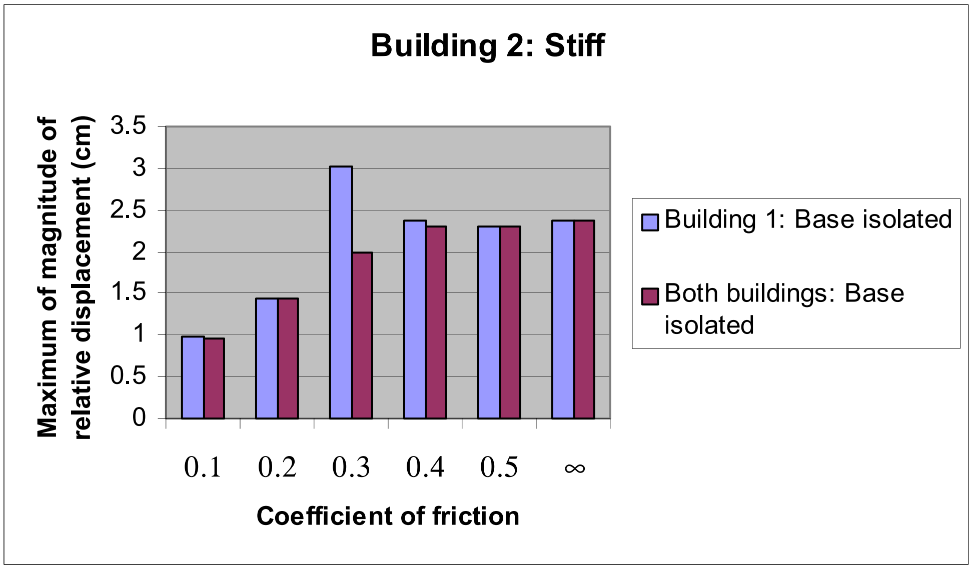

Fig 4.16 Maximum relative displacement as a function of base isolation friction coefficient with stiff Building 2

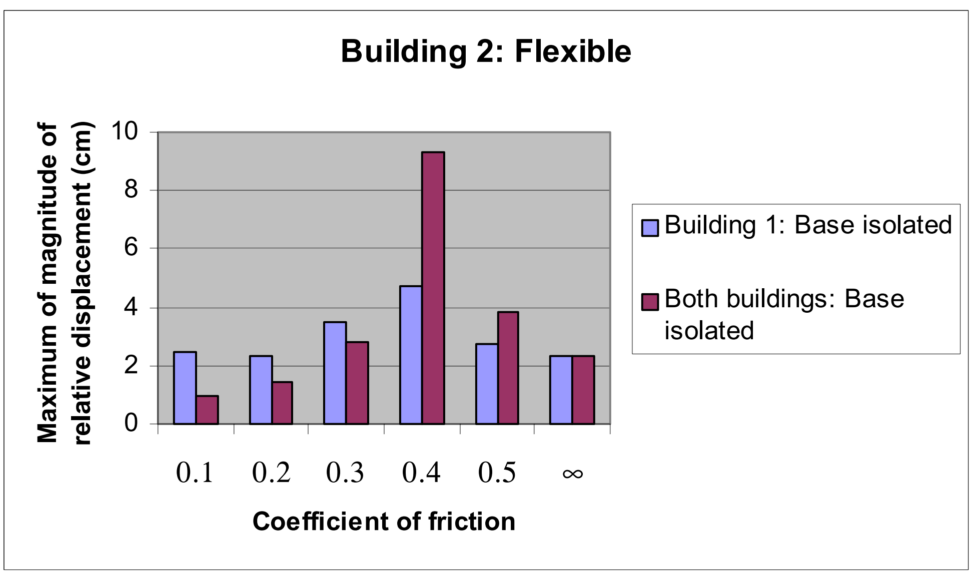

Fig 4.17 Maximum relative displacement as a function of base isolation friction coefficient with flexible Building 2

Bar graphs for relative displacements of second floor of Building 1 for El Centro earthquake have been given in figures 4.16 and 4.17. Note that the relative displacement is the displacement of a building with respect to its base and does not include the sliding displacement. It can be seen from these graphs that as expected the relative displacements decreased with decrease in coefficient of friction or increase in sliding. From figure 4.16, it can be seen that there is not much difference in response between single base isolated and both base isolated cases for stiff Building 2 but the response is considerably less for both base isolated case when Building 2 is flexible as is visible from figure 4.17. In the later case the response again starts decreasing, since, with increase in coefficient of friction, sliding decreases, which increases the relative response and increases the pounding. This pounding decreases the response of the system.

Table 4.5 shows some of the statistics for total displacement with varying friction coefficients in the base isolation system and stiff Building 2. The gap between the adjacent buildings is equal to 2 centimeters. Table 4.6 shows the same statistics for different earthquakes and when friction coefficient equal to 0.2. Here also the gap between the buildings is kept 4 cm and 8 cm respectively when subjected to Loma Prieta and Northridge earthquakes. Table 4.7 and 4.8 are the corresponding tables when Building 2 is flexible. Comparing the means of response in tables 4.6 and 4.8, it can be seen that there is not much difference in the values. This happens since both the buildings can slide in this case keeping the gap between them approximately constant. It was observed that in this case no pounding took place for Northridge earthquake and very little for Loma Prieta earthquake. For El Centro earthquake, some pounding took place but the impact force was not large enough to cause sliding. A similar phenomenon can be observed in table 4.5 and 4.7 where the buildings are subjected to El Centro earthquake.

Table 4.5 Response behavior characterization for varying friction coefficients. Both buildings: base isolated, Building 2: stiff

Adjacent buildings with stiff Building 2							
Statistics	Single reference building, fixed base	Friction coefficients for both base isolated buildings					
		$\mu = \infty$	$\mu = 0.5$	$\mu = 0.4$	$\mu = 0.3$	$\mu = 0.2$	$\mu = 0.1$
Mean (cm)	0	-0.0021	0.46	-0.033	0.88	0.64	0.58
Variance (cm)	0.49	0.4356	0.3721	0.3721	0.3844	0.2809	0.2025
Std deviation (cm)	0.7	0.66	0.61	0.61	0.62	0.53	0.45
Skewness	0.0246	-0.0107	-0.2036	0.1803	-0.9856	-0.2221	-1.2491
CoE	1.8417	0.9556	0.7537	0.5925	1.0621	0.9919	2.7623
Total number of impacts	NA	8	16	4	22	18	1

Table 4.6 Response behavior characterization for earthquakes of increasing intensity. Both buildings: base isolated, Building 2: stiff

Friction coefficeint μ = 0.2 , Gap = 2 cm						
Earthquake	Mean (cm)	Std deviation (cm)	Skewness	CoE	Inphase impacts	Out of phase impacts
Mexico City	0	0.07486	0.1465	4.7465	0	0
El Centro	0.64	0.53	-0.2221	0.9919	4	14
Loma Prieta, Gap = 4 cm	-2.05	1.06	1.2448	0.6232	0	0
Northridge, Gap = 8 cm	2.95	2.99	-0.8068	0.925	0	0

Table 4.7 Response behavior characterization for varying friction coefficients. Both buildings: base isolated, Building 2: flexible

Adjacent buildings with flexible Building 2							
Statistics	Single reference building, fixed base	Friction coefficients for both base isolated buildings					
		$\mu = \infty$	$\mu = 0.5$	$\mu = 0.4$	$\mu = 0.3$	$\mu = 0.2$	$\mu = 0.1$
Mean (cm)	0	-0.027	1.62	-8.55	-0.17	0.26	0.47
Variance (cm)	0.49	0.16	0.6889	20.25	0.3481	0.2809	0.1681
Std deviation (cm)	0.7	0.4	0.83	4.5	0.59	0.53	0.41
Skewness	0.0246	-0.1577	-1.9928	1.31	0.6325	0.0264	-0.5834
CoE	1.8417	2.8934	6.1115	-0.1012	1.0328	0.1762	2.4457
Total number of impacts	NA	38	47	20	15	2	3

Table 4.8 Response behavior characterization for earthquakes of increasing intensity. Both buildings: base isolated, Building 2: flexible

Friction coefficient μ = 0.2, Gap = 2 cm						
Earthquake	Mean (cm)	Std deviation (cm)	Skewness	CoE	Inphase impacts	Out of phase impacts
Mexico City	0	0.07486	0.1465	4.7465	0	0
El Centro	0.26	0.53	0.0264	0.1762	0	2
Loma Prieta, Gap = 4 cm	-1.86	0.95	1.305	1.0618	0	2
Northridge, Gap = 8 cm	3.01	3.02	-0.7647	0.7906	0	0

Figure 4.18 presents four histograms and normal probability plots of total displacement response of second floor of Building 1 for adjacent buildings subjected to El Centro earthquake. It can be seen that with a stiff Building 2 and base isolated Building 1, the mean is on the positive side but when Building 2 is flexible, the mean shifts to negative since pounding with the flexible building caused large sliding displacements in negative direction. In this case if flexible Building 2 is also base isolated, then it can be seen that the mean again shifts to positive side since there is considerable reduction in response of the flexible building due to base isolation which in turn reduces pounding. Thus in case of flexible Building 2, response is governed by pounding since impact force is high. For stiff Building 2, if both the buildings are base isolated, then there is not a considerable change in response since the response is dominated by earthquake, as the impact force of pounding is very low.

Similar discussion holds for adjacent buildings whose response histograms are shown in figure 4.19. Here the buildings are subjected to Loma Prieta earthquake, which is a considerably different earthquake than El Centro earthquake as shown in section 4.1. In this case also the sliding displacement of Building 1 is considerably reduced when Building 2 is made base isolated. This phenomenon is also well reflected from the time histories of the response given in figure 4.20. Thus the worst case can be seen as the one when base isolated building is close to fixed base building and one or both of them are flexible. In this case the impact forces are large and pounding if occurs can cause lot of sliding displacements in addition to local damage.

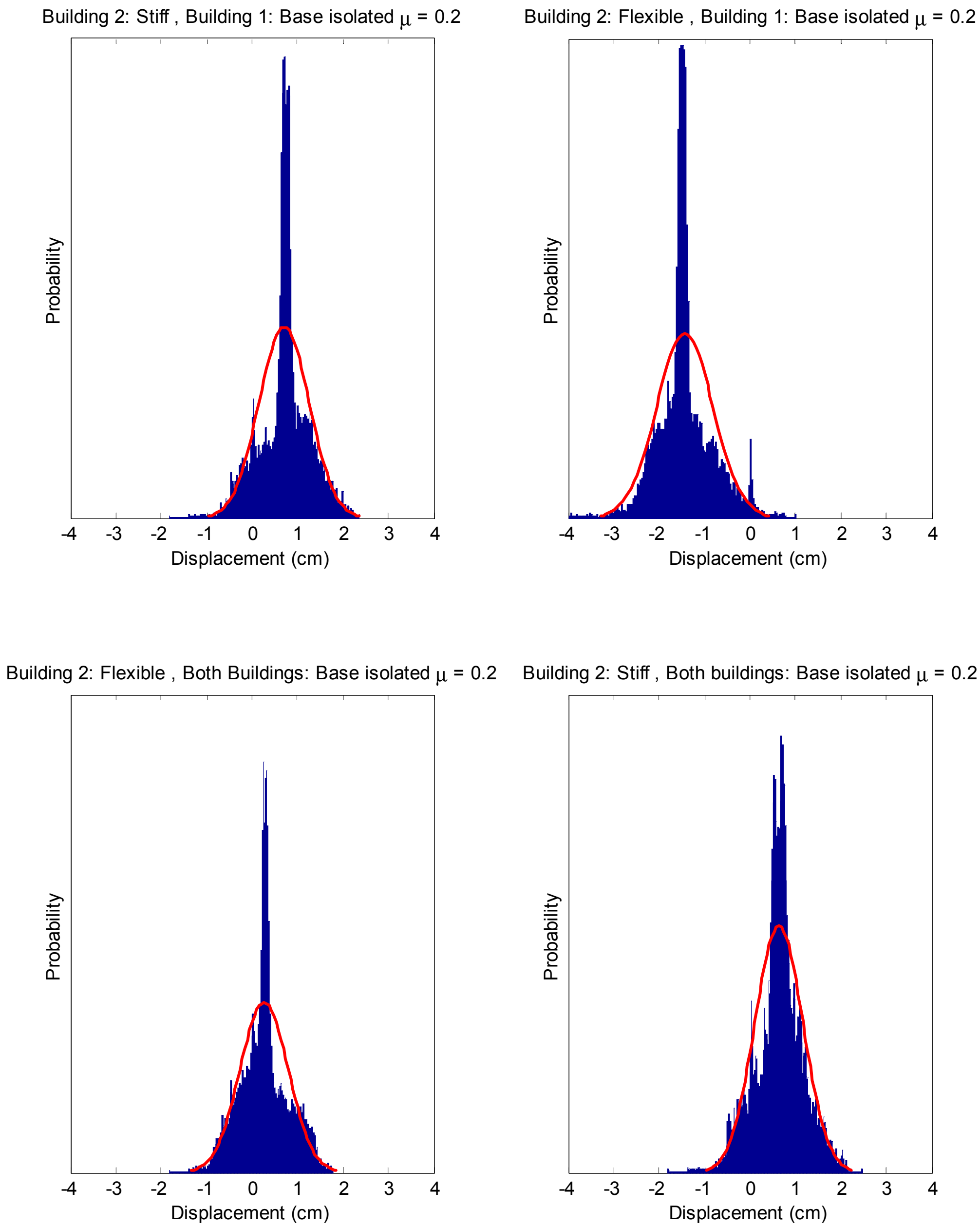

Fig 4.18 Histogram and normal probability density function of total displacement for El Centro earthquake, Gap = 2 cm

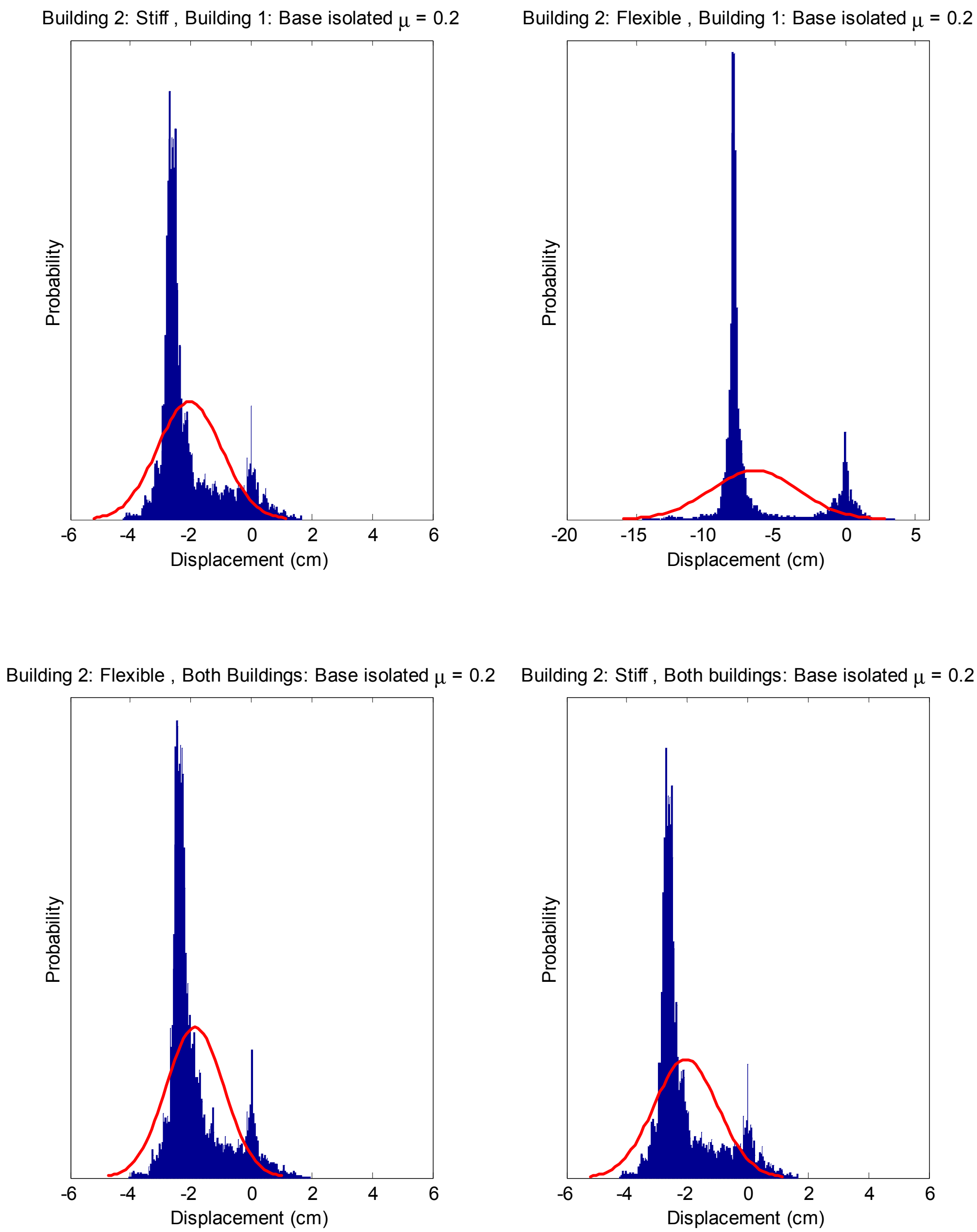

Fig 4.19 Histogram and normal probability density function of total displacement for Loma Prieta earthquake, Gap = 4 cm

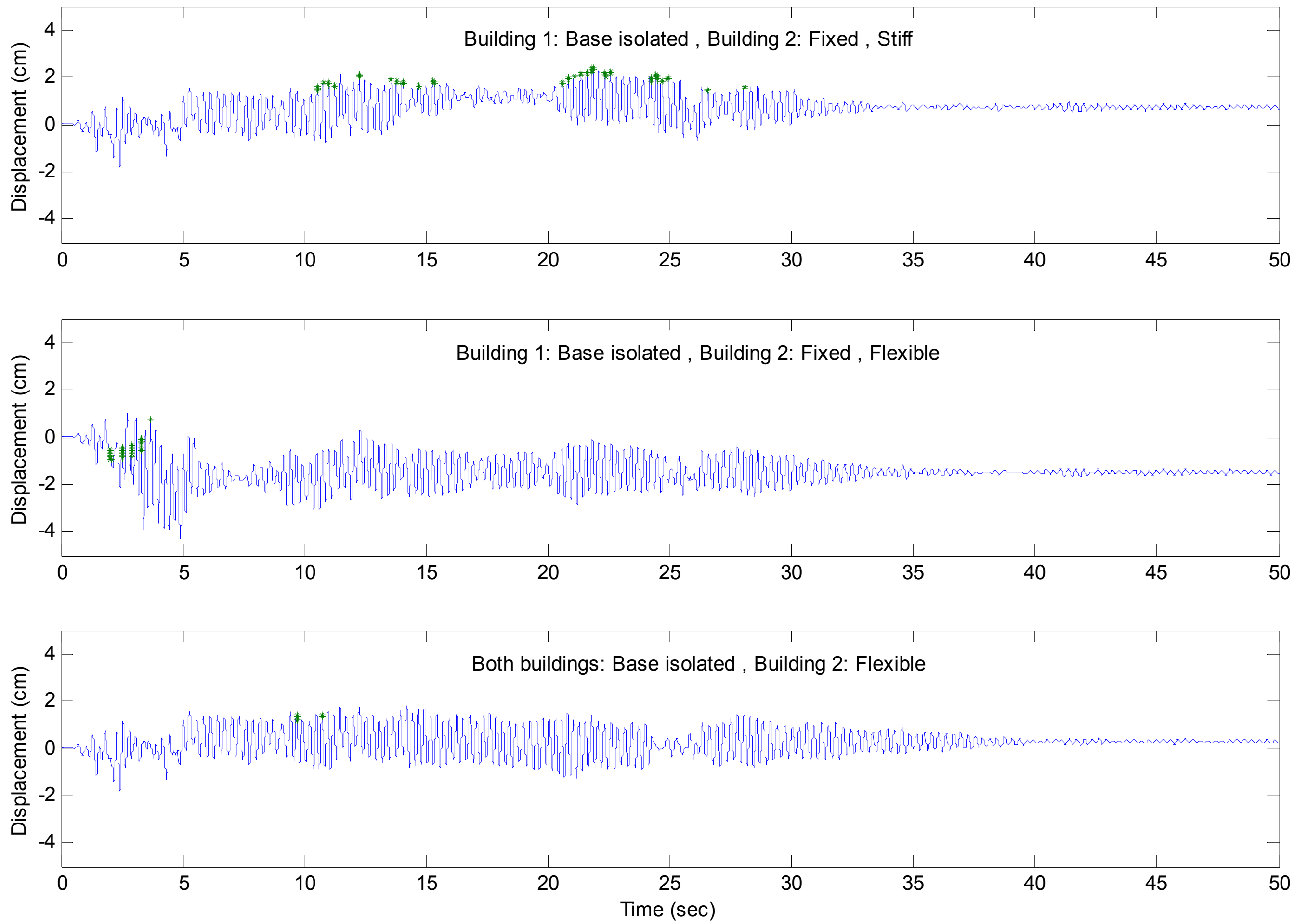

Fig 4.20 Time histories of total displacement for El Centro earthquake, μ = 0.2, Gap = 2 cm

The correlation coefficients of the displacement response of Building 1 for various cases of base isolation and for stiff Building 2 to single fixed building response are presented in table 4.9 and table 4.10 shows the corresponding correlation coefficients for flexible Building 2. It can be observed that for fixed-fixed case, correlation coefficients are lower for flexible Building 2, since there is more pounding in flexible buildings as discussed earlier. Since both pounding and sliding are non-linear phenomenon, the correlation coefficients are very small. Correlation between the responses decreases when pounding is introduced and further decreases as base isolation effect is increased. Due to little correlation between the responses, it was earlier found difficult to anticipate the variation of parameters like number of impact, their magnitude, in-phase and out of phase impacts, duration of impacts, maximum total sliding displacement, relative displacement, sliding displacement, residual sliding displacements and statistical parameters discussed earlier, as a function of coefficient of friction in base isolation.

In this study no centering force was considered in the base isolation system and also there is no limit on the maximum sliding displacement. In an actual base isolation system there will be a limit on the maximum sliding displacement. Also the friction pendulum system that is a more commonly used for friction bearing base isolation will also have a centering force. Adding a maximum limit on sliding would have added another non-linear effect and would have made the response more unpredictable and difficult to interpret. Both of these factors tend to maintain gap between the buildings thus reducing the chances of pounding, which occurred in our simulations.

Table 4.9 Correlation coefficients, Building 2: stiff

μ	Fixed-Base isolated	Base isolated-Base isolated
Fixed – Fixed, $\mu = \infty$	0.9529	0.9529
0.3	0.0661	0.3404
0.2	-0.0548	-0.0137
0.1	0.0378	0.027

Table 4.10 Correlation coefficients, Building 2: flexible

μ	Fixed-Base isolated	Base isolated-Base isolated
Fixed – Fixed, $\mu = \infty$	0.4468	0.4468
0.3	0.2208	0.4476
0.2	0.0257	-0.0156
0.1	0.0161	0.0064

5. SUMMARY AND CONCLUSION

5.1. SUMMARY AND SCOPE OF STUDY

In this research investigation, adjacent buildings were modeled as lumped mass systems with impact taking place only at floor levels. Inelastic impacts were taken into account by utilizing linear elastic spring and dashpots at the floor level where the impact occurs, as was suggested earlier by Anagnostopoulos and Spiliopoulos (1992). Base isolation for each building was simulated by allowing the building to slide along a horizontal frictional plane at a specified foundation elevation (Mostaghel and Tanbakuchi 1983). The base isolation system adopted in this study is flat sliding surface type and no centering force acts at any time nor is there a limit on the maximum sliding displacement by the buildings. Of course for actual systems both of these considerations must be addressed. Friction coefficients between the sliding surfaces have also been assumed constant which otherwise would vary in practice. Inelastic behavior and any rotational affects of the buildings have been neglected.

The dynamic response equations written for each building include effects of sliding and building impact. The resulting systems of second order constant coefficient equations are recast as a system of first order ordinary differential equations and solved using 'MATLAB' ode solvers. The external loading on the structures includes actual acceleration time histories from earthquakes in California and Mexico. The equations were formulated so that the adjacent structures can have different foundations (fixed or base isolated) and can be subjected to different ground motions. In this analysis, buildings are modeled as shear buildings and do not include frame behavior. Further, damage caused by building impacts has not been taken into account and this can make the model prediction unrealistic. The spacing between the buildings in some cases was kept small in order to study pounding and impact. Although current design codes call for larger spacing between base isolated buildings, in some cases like historic restoration and seismic rehabilitation of old fixed base buildings using base isolation systems, the gap may indeed become quite small.

In this research investigation, the focus of the numerical simulation is two degree of freedom system. Three adjacent building configurations were investigated in the process of studying building pounding. The first configuration is the common one in which both the adjacent buildings are fixed base. The second configuration investigates the pounding between a base isolated and a fixed base building and in the final configuration, both of the adjacent buildings are base isolated. Pounding between the floors of adjacent buildings can occur when the buildings vibrate in modes that are not completely in-phase. Building pounding can influence the response of the colliding buildings and can cause local damage at the locations where impact takes place. For base isolated building systems, pounding behavior can be favorably or unfavorably modified depending upon the nature of ground motion and sliding friction.

5.2. CONCLUSIONS

When both of the buildings have fixed base, the analysis showed that there is a reduction in the number of impacts as the gap between the buildings is increased. This reduction in the number of impacts is much faster when the engaging buildings are stiff. It was found that the magnitude of impact is higher when the buildings are flexible and decreases for both stiff and flexible building cases with an increase in the gap spacing. In this study it is possible to classify the pounding as resulting from in-phase and out of phase impacts. In-phase impacts are those that occur while the buildings are moving in the same direction and out of phase impacts are those that occur when the buildings are moving in the opposite direction and towards each other. It was observed that out of phase motions decreases the response of the system while in-phase motions can amplify it. Further the impact force is much higher in out of phase impacts as would be expected, since the relative velocity between the buildings is higher. This then can result in significant local damage and lead to a decrease in the building displacements. It was observed that number of out of phase impacts were more than the in-phase impacts. A reduction in the response of buildings was found due to pounding, probably due to higher number of out of phase impacts. Also an increase in the peak displacement response was found when the gap between the buildings is very small. This peak displacement was higher when more flexible building is one of the adjacent buildings. This is in compliance with the study done by Anagnostopoulos 1988, who reached the same conclusion for fixed base buildings with very small gap.

In the second and third configurations where one or both the buildings are base isolated, the problem of pounding will generally be less since there is a significant reduction in the relative displacement response due to base isolation. Pounding in buildings with base isolated systems can occur once they reach maximum allowable sliding displacements. Thus probability of pounding in adjacent buildings is even lower when both the buildings are base isolated since during ground motion they will tend to slide in the same direction and there is very low probability that they will be at there nearest edges of maximum sliding limits at the same time. The chances of pounding is more if only one of the buildings is base isolated, since in this case the base isolated building can slide towards the fixed base building reducing the gap between them.

If pounding occurs in base isolated buildings it can be harmful, since, in addition to other disadvantages, the impact forces that act during pounding can cause additional sliding of the base isolated buildings. It was found that impact forces are much higher when one or both of the colliding buildings are flexible, similar to the fixed base buildings. Further, when only one of the adjacent buildings is base isolated and one or both of the buildings are flexible is the worst. In this case the chances of pounding are high and so is the impact force, which can cause very large sliding displacements of the base isolated building.

When only one of the buildings is base isolated, then the amount of pounding depends on the placement of the buildings with respect to the earthquake. For example, when subjected to El Centro earthquake, the base isolated building tends to have a positive sliding displacement and thus increases the amount of pounding with Building 2. Whereas when subjected to Loma Prieta earthquake, the base isolated building have a negative sliding displacement that increases the gap and reduces pounding. When both the buildings are base isolated, probability of pounding can increase if there is a lot of mass difference between the buildings. This causes the buildings to slide differently and thus reducing the gap between them in some cases.

5.3. RECOMMENDATIONS FOR FURTHER STUDY

Further studies should take into account the centering force and should address constraints such as the maximum limit on the sliding displacement in base isolation system. These factors were not considered in this study since it was the first attempt to know the behavior of pounding in base isolated buildings and introducing them would have made the response more unpredictable and difficult to interpret. Both of these factors tend to maintain the gap between the buildings, thus reducing the chances of pounding. Other parametric studies can also be pursued varying the stiffness of the lumped mass model. More specifically, changing the stiffness of Building 2, instead of varying the lumped masses of the floors as done in the present study. The results obtained could be compared with those obtained in this study.

Further studies on research in in-phase and out of phase impacts is needed. Systems of adjacent buildings can be identified (for their time periods) so that in-phase and out of phase impacts between them and their effects are within specified allowable limits. SPD method (Kasai *et al* 1996) and CQC method (Penzien 1997) that take into account vibration phase can be used to supplement this study as well as studying the applicability of the SRSS method to base isolated buildings. Separate specifications for gap between adjacent base isolated and fixed base buildings and adjacent base isolated buildings are recommended since these arrays of buildings behave differently during ground motion.

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APPENDIX 1

MATLAB IMPLEMENTATION OF DYNAMIC EQUATION FOR THREE ADJACENT SINGLE DEGREE OF FREEDOM BASE ISOLATED BUILDINGS

Base isolation system, like pounding phenomenon, has a non-linear behavior and thus problems involving them should be solved numerically. While solving numerically, the solution after every time step has to be analyzed to see what preset criterion does the system follows and the system properties has to be changed accordingly. MATLAB has been used to solve the dynamic equations. Since MATLAB can solve first order equations, we need to convert the dynamic equation that is second order into first order. To do this we make the following assumptions.

$$\dot{x}_{11,B11} = y_1,\ \ \dot{x}_{21,B21} = y_2,\ \ \dot{x}_{31,B31} = y_3,\ \ \dot{x}_{B11,B10} = y_4,$$

$$\dot{x}_{B21,B20} = y_5,\ \ \dot{x}_{B31,B30} = y_6,\ \ x_{11,B11} = y_7,\ \ x_{21,B21} = y_8,$$

$$x_{31,B31} = y_9,\ \ x_{B11,B10} = y_{10},\ \ x_{B21,B20} = y_{11},\ \ x_{B31,B30} = y_{12}$$

Substituting in the dynamic equation for structure (2.36) and sliding (2.38), (2.39) and (2.40), we obtain first order equations. These substitutions are also done in the conditions for pounding (2.37) and sliding (2.41), (2.42) and (2.43). Using these one can also deduce six additional equations

$$\dot{y}_7 - y_1 = 0,\ \ \dot{y}_8 - y_2 = 0,\ \ \dot{y}_9 - y_3 = 0,$$

$$\dot{y}_{10} - a_1 y_4 = 0,\ \ \dot{y}_{11} - a_2 y_5 = 0,\ \ \dot{y}_{12} - a_3 y_6 = 0$$

Where, the parameter a_i $(i = 1,2,3)$ are defined to account for sliding in each of the buildings while executing the computer program. Their value equals to 1 for sliding and 0 for no sliding. These parameters can be updated after each iteration and for each of the buildings separately. a_i equals to zero implies zero sliding velocity and acceleration for the ith building. The dynamic equation for structure, for sliding and the six additional equations can be combined to form 12×12 matrix equation, rearranged and written in a simplified form.

$$[M_1]\{\dot{Y}\}+([C_1]+[C_2])\{Y\}+([K_1]+[K_2])\{Y\}+[C_3]\{Y\}+[K_3]\{Y\} \\ +\{D_1\}-[D_2]\ \{Y\}=\{F\}-[M_2]\{\dot{Y}\}-[M_3]\{\ddot{X}_G\}$$

This equation can be rearranged to a form compatible with that required for 'ode' solver of the MATLAB also known as the state space representation. This equation along with the conditions for pounding and sliding are solved using the 'ode45' solver of the MATLAB. The computer programs written to accomplish this task have been included in Appendix 3. The state space repesentation is

$$\{\dot{Y}\}=\left(\ [M_1]+[M_2]\ \right)^{-1}\left(\ \{F\}-[M_3]\{\ddot{X}_G\} \\ -(\ [C_1]+[C_2]+[C_3]+[K_1]+[K_2]+[K_3]-[D_2]\ )\{Y\}-\{D_1\}\ \right)$$

Where,

$$[M_1]=\begin{bmatrix} m_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{31} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_1m_{11} & 0 & 0 & m_{11}+M_{B11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_2m_{21} & 0 & 0 & m_{21}+M_{B21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_3m_{31} & 0 & 0 & m_{31}+M_{B31} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[M_2]=\begin{bmatrix}0&0&0&m_{11}&0&0&0&0&0&0&0&0\\0&0&0&0&m_{21}&0&0&0&0&0&0&0\\0&0&0&0&0&m_{31}&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\end{bmatrix}$$

$$[M_3]=\begin{bmatrix}m_{11}&0&0&0&0&0&0&0&0&0&0&0\\0&m_{21}&0&0&0&0&0&0&0&0&0&0\\0&0&m_{31}&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\end{bmatrix}$$

$$[C_1]=\begin{bmatrix} c_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{31} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[C_2]=\begin{bmatrix} c_{11,21} & -c_{11,21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -c_{11,21} & c_{11,21}+c_{21,31} & -c_{21,31} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -c_{21,31} & c_{21,31} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[C_3]=\begin{bmatrix} 0&0&0&c_{11,21}&-c_{11,21}&0&0&0&0&0&0&0\\ 0&0&0&-c_{11,21}&c_{11,21}+c_{21,31}&-c_{21,31}&0&0&0&0&0&0\\ 0&0&0&0&-c_{21,31}&c_{21,31}&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$[K_1]=\begin{bmatrix} 0&0&0&0&0&0&k_{11}&0&0&0&0&0\\ 0&0&0&0&0&0&0&k_{21}&0&0&0&0\\ 0&0&0&0&0&0&0&0&k_{31}&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$[K_2]=\begin{bmatrix} 0&0&0&0&0&0&s_{11,21}&-s_{11,21}&0&0&0&0\\ 0&0&0&0&0&0&-s_{11,21}&s_{11,21}+s_{21,31}&-s_{21,31}&0&0&0\\ 0&0&0&0&0&0&0&-s_{21,31}&s_{21,31}&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$[K_3]=\begin{bmatrix} 0&0&0&0&0&0&0&0&0&s_{11,21}&-s_{11,21}&0\\ 0&0&0&0&0&0&0&0&0&-s_{11,21}&s_{11,21}+s_{21,31}&-s_{21,31}\\ 0&0&0&0&0&0&0&0&0&0&-s_{21,31}&s_{21,31}\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$\{D_1\}=\begin{bmatrix}-s_{11,21}d_{11,21} & s_{11,21}d_{11,21}-s_{21,31}d_{21,31} & s_{21,31}d_{21,31} & 0&0&0&0&0&0&0&0&0\end{bmatrix}^T$$

$$[D_2]=\begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&a_1&0&0&0&0&0&0&0&0\\ 0&0&0&0&a_2&0&0&0&0&0&0&0\\ 0&0&0&0&0&a_3&0&0&0&0&0&0 \end{bmatrix}$$

$$\{F\}=\begin{bmatrix} F_{11}\\ F_{21}\\ F_{31}\\ a_1(-\mathrm{sgn}(y_4))F_{f_1}+a_1F_{11}-a_1(m_{11}+M_{B11})\ddot{x}_{B10}\\ a_2(-\mathrm{sgn}(y_5))F_{f_2}+a_2F_{21}-a_2(m_{21}+M_{B21})\ddot{x}_{B20}\\ a_3(-\mathrm{sgn}(y_6))F_{f_3}+a_3F_{31}-a_3(m_{31}+M_{B31})\ddot{x}_{B30}\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0 \end{bmatrix}$$

$$\{\dot{Y}\}=\begin{bmatrix}\dot{y}_1 & \dot{y}_2 & \dot{y}_3 & \dot{y}_4 & \dot{y}_5 & \dot{y}_6 & \dot{y}_7 & \dot{y}_8 & \dot{y}_9 & \dot{y}_{10} & \dot{y}_{11} & \dot{y}_{12}\end{bmatrix}^T$$

$$\{Y\}=\begin{bmatrix}y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & y_7 & y_8 & y_9 & y_{10} & y_{11} & y_{12}\end{bmatrix}^T$$

APPENDIX 2

MATLAB IMPLEMENTATION OF DYNAMIC EQUATION FOR TWO ADJACENT TWO DEGREE OF FREEDOM BASE ISOLATED BUILDINGS

Base isolation system, like pounding phenomenon, has a non-linear behavior and thus problems involving them should be solved numerically. While solving numerically, the solution after every time step has to be analyzed to see what preset criterion does the system follows and the system properties has to be changed accordingly. MATLAB has been used to solve the dynamic equations. Since MATLAB can solve first order equations, we need to convert the dynamic equation that is second order into first order. To do this we make the following assumptions.

$$\dot{x}_{11,B11} = y_1,\quad \dot{x}_{12,B11} = y_2,\ \dot{x}_{21,B21} = y_3,\ \dot{x}_{22,B21} = y_4,$$

$$\dot{x}_{B11,B10} = y_5,\ \dot{x}_{B21,B20} = y_6,\ x_{11,B11} = y_7,\ x_{12,B11} = y_8,$$

$$x_{21,B21} = y_9,\ x_{22,B21} = y_{10},\ x_{B11,B10} = y_{11},\ x_{B21,B20} = y_{12}$$

Substituting in the dynamic equation for structure (2.47) and sliding (2.49) and (2.50), we obtain first order equations. These substitutions are also done in the conditions for pounding (2.48) and sliding (2.51) and (2.52). Using these one can also deduce six additional equations

$$\dot{y}_7 - y_1 = 0,\ \dot{y}_8 - y_2 = 0,\ \dot{y}_9 - y_3 = 0,$$

$$\dot{y}_{10} - y_4 = 0,\ \dot{y}_{11} - a_1 y_5 = 0,\ \dot{y}_{12} - a_2 y_6 = 0$$

Where, the parameter a_i $(i = 1,2)$ are defined to account for sliding in each of the buildings while executing the computer program. Their value equals to 1 for sliding and 0 for no sliding. These parameters can be updated after each iteration and for each of the buildings separately. a_i equals to zero implies zero sliding velocity and acceleration for the ith building. The dynamic equation for structure, for sliding and the six additional equations can be combined to form 12×12 matrix equation, rearranged and written in a simplified form.

$$[M_1]\{\dot{Y}\}+([C_1]+[C_2])\{Y\}+([K_1]+[K_2])\{Y\}+[C_3]\{Y\}+[K_3]\{Y\} \\ +\{D_1\}-[D_2]\ \{Y\}=\{F\}-[M_2]\{\dot{Y}\}-[M_3]\{\ddot{X}_G\}$$

This equation can be further rearranged to a form compatible with that required for 'ode' solver of the MATLAB also known as the state space representation. This equation along with the conditions for pounding and sliding are solved using the 'ode45' solver of the MATLAB. The computer programs written to accomplish this task have been included in Appendix 4. The state space representation is

$$\{\dot{Y}\}=\left(\ [M_1]+[M_2]\ \right)^{-1}\left(\ \{F\}-[M_3]\{\ddot{X}_G\} \right. \\ \left. -(\ [C_1]+[C_2]+[C_3]+[K_1]+[K_2]+[K_3]-[D_2]\ )\{Y\}-\{D_1\}\ \right)$$

Where,

$$[M_1]=\begin{bmatrix} m_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_1 m_{11} & a_1 m_{12} & 0 & 0 & m_{11}+m_{12}+M_{B11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_2 m_{21} & a_2 m_{22} & 0 & m_{21}+m_{22}+M_{B21} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[M_2]=\begin{bmatrix} 0&0&0&0&m_{11}&0&0&0&0&0&0&0\\ 0&0&0&0&m_{12}&0&0&0&0&0&0&0\\ 0&0&0&0&0&m_{21}&0&0&0&0&0&0\\ 0&0&0&0&0&m_{22}&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$[M_3]=\begin{bmatrix} m_{11}&0&0&0&0&0&0&0&0&0&0&0\\ 0&m_{12}&0&0&0&0&0&0&0&0&0&0\\ 0&0&m_{21}&0&0&0&0&0&0&0&0&0\\ 0&0&0&m_{22}&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$[C_1]=\begin{bmatrix} c_{11}+c_{12} & -c_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -c_{12} & c_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{21}+c_{22} & -c_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -c_{22} & c_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[C_2]=\begin{bmatrix} c_{11,21} & 0 & -c_{11,21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{12,22} & 0 & -c_{12,22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -c_{11,21} & 0 & c_{11,21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -c_{12,22} & 0 & c_{12,22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[C_3]=\begin{bmatrix}0&0&0&0&c_{11,21}&-c_{11,21}&0&0&0&0&0&0\\0&0&0&0&c_{12,22}&-c_{12,22}&0&0&0&0&0&0\\0&0&0&0&-c_{11,21}&c_{11,21}&0&0&0&0&0&0\\0&0&0&0&-c_{12,22}&c_{12,22}&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\end{bmatrix}$$

$$[K_1]=\begin{bmatrix}0&0&0&0&0&0&k_{11}+k_{12}&-k_{12}&0&0&0&0\\0&0&0&0&0&0&-k_{12}&k_{12}&0&0&0&0\\0&0&0&0&0&0&0&0&k_{21}+k_{22}&-k_{22}&0&0\\0&0&0&0&0&0&0&0&-k_{22}&k_{22}&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\end{bmatrix}$$

$$[K_2]=\begin{bmatrix} 0&0&0&0&0&0&s_{11,21}&0&-s_{11,21}&0&0&0\\ 0&0&0&0&0&0&0&s_{12,22}&0&-s_{12,22}&0&0\\ 0&0&0&0&0&0&-s_{11,21}&0&s_{11,21}&0&0&0\\ 0&0&0&0&0&0&0&-s_{12,22}&0&s_{12,22}&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$[K_3]=\begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&s_{11,21}&-s_{11,21}\\ 0&0&0&0&0&0&0&0&0&0&s_{12,22}&-s_{12,22}\\ 0&0&0&0&0&0&0&0&0&0&-s_{11,21}&s_{11,21}\\ 0&0&0&0&0&0&0&0&0&0&-s_{12,22}&s_{12,22}\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$\{D_1\}=\begin{bmatrix} -s_{11,21}d_{11,21} & -s_{12,22}d_{12,22} & s_{11,21}d_{11,21} & s_{12,22}d_{12,22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$[D_2] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_2 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\{F\} = \begin{bmatrix} F_{11} \\ F_{12} \\ F_{21} \\ F_{22} \\ a_1(-\operatorname{sgn}(y_5))F_{f_1} + a_1(F_{11}+F_{12}) - a_1(m_{11}+m_{12}+M_{B11})\ddot{x}_{B10} \\ a_2(-\operatorname{sgn}(y_6))F_{f_2} + a_2(F_{21}+F_{22}) - a_2(m_{21}+m_{22}+M_{B21})\ddot{x}_{B20} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\{\dot{Y}\} = \begin{bmatrix} \dot{y}_1 & \dot{y}_2 & \dot{y}_3 & \dot{y}_4 & \dot{y}_5 & \dot{y}_6 & \dot{y}_7 & \dot{y}_8 & \dot{y}_9 & \dot{y}_{10} & \dot{y}_{11} & \dot{y}_{12} \end{bmatrix}^T$$

$$\{Y\} = \begin{bmatrix} y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & y_7 & y_8 & y_9 & y_{10} & y_{11} & y_{12} \end{bmatrix}^T$$

APPENDIX 3

COMPUTER PROGRAMS IN MATLAB TO OBTAIN RESPONSE OF THREE ADJACENT SINGLE DEGREE OF FREEDOM BASE ISOLATED BUILDINGS

```
% 'MAIN' FUNCTION
% THIS PROGRAM CALCULATES THE RESPONSE OF THREE ADJACENT 1-DOF BASE
% ISOLATED SYSTEMS. THE OUTPUT RESPONSE IS STORED IN THE VECTOR 'RESULT'
% WHICH HAVE NINTEEN COLUMNS. THE FIRST COLUMN IS THE TIME VECTOR AND
% OTHER EIGHTEEN COLUMNS CONTAINS THE ACCELERATION, VELOCITY AND
% DISPLACEMENT RESPONSES
%
% INPUT COMMAND IS 'main(h,to)'
% WHERE h = TIME STEP AND to = MAXIMUM TIME OF SIMULATION
%
% OTHER PROGRAMS WHICH IT CALLS ARE 'solver', 'f', 'wind_force', 'elcentro_data'
%
% INPUT PARAMETERS LIKE THE STRUCTURAL MASS, STIFFNESS AND DAMPING, IMPACT
% STIFFNESS AND DASHPOTS, SPACING BETWEEN THE BUILDINGS, MASS OF THE BASE
% OF STRUCTURE AND BASE ISOLATION FRICTION COEFFICIENT SHOULD BE CHANGED
% WITHIN THE PROGRAM.
%
% ALL INPUTS ARE IN SI UNITS
%
% IN THIS PROGRAM 'AbsTol' HAS BEEN REDUCED TO 1e-3
%
% WRITTEN BY
%
% VIVEK KUMAR AGARWAL, SPRING 2004

function main(h,to)

% INPUT PARAMETERS-----------------------------------------------------------------------------------------------

% ALL PARAMETERS IN SI UNITS

m11 = 17511.8;      % STRUCTURAL MASS
m21 = 35023.6;
m31 = 17511.8;

c11 = 12363.33;     % STRUCTURAL DAMPING
c21 = 17511.8;
c31 = 12363.33;

k11 = 875590.55;    % STRUCTURAL STIFFNESS
k21 = 875590.55;
k31 = 875590.55;

damping1121 = 122805;       % IMPACT DASHPOT BETWEEN BUILDINGS 1 & 2
damping2131 = 122805;
stiffness1121 = 17511811;   % IMPACT STIFFNESS BETWEEN BUILDINGS 1 & 2
stiffness2131 = 17511811;
d1121 = 0.1;                % SPACING BETWEEN BUILDINGS 1 & 2
d2131 = 0.1;

Mb11 = 1.58;     % MASS OF THE BASE OF STRUCTURE
Mb21 = 1.58;
Mb31 = 1.58;
```

```
mu1 = 50.05;     % FRICTION COEFFICIENT IN BASE ISOLATION
mu2 = 50.05;
mu3 = 50.05;


g = 9.814;       % ACCELEATION DUE TO GRAVITY

Ff1 = mu1*(m11+Mb11)*g;
Ff2 = mu2*(m21+Mb21)*g;
Ff3 = mu3*(m31+Mb31)*g;

% INITIAL CONDITIONS---------------------------------------------------------------------------------------------
j = 1;
t1 = 0;
time = 0;
Xoog = 0;
f11 = 0;
f21 = 0;
f31 = 0;

Xoo11b11 = 0;   % INITIAL CONDITIONS FOR BUILDING 1
Xo11b11 = 0;
X11b11 = 0;
Xoob11b10 = 0;
Xob11b10 = 0;
Xb11b10 = 0;

Xoo21b21 = 0;   % INITIAL CONDITIONS FOR BUILDING 2
Xo21b21 = 0;
X21b21 = 0;
Xoob21b20 = 0;
Xob21b20 = 0;
Xb21b20 = 0;

Xoo31b31 = 0;   % INITIAL CONDITIONS FOR BUILDING 3
Xo31b31 = 0;
X31b31 = 0;
Xoob31b30 = 0;
Xob31b30 = 0;
Xb31b30 = 0;

X11b10 = 0;
X21b20 = 0;
X31b30 = 0;

a1 = 0;
a2 = 0;
a3 = 0;

result(j,:) = [time  Xoo11b11  Xo11b11  X11b11  Xoob11b10  Xob11b10  Xb11b10  Xoo21b21  Xo21b21
X21b21  Xoob21b20  Xob21b20  Xb21b20  Xoo31b31  Xo31b31  X31b31  Xoob31b30  Xob31b30
Xb31b30];
```

```
%           1      2      3      4      5      6      7      8      9     10     11      12      13     14
% 15      16     17      18      19
% COLUMN NUMBER

% Initialize j12, impact_time1121(j12,:), j23, impact_time2131(j23,:)
j12 = 1;
impact_time1121(j12,:) = [0 0 0];
j23 = 1;
impact_time2131(j23,:) = [0 0 0];

% 'FOR' LOOP TO CALCULATE FOR EACH TIME STEP-----------------------------------------------------

for t2 = h:h:to

   % CHECK FOR POUNDING-------------------------------------------------------------------------------------

   if X11b10 - X21b20 - d1121 <= 0
     s1121 = 0; c1121 = 0;        % NO POUNDING BETWEEN BUILDINGS 1 & 2
   else
     s1121 = stiffness1121; c1121 = damping1121;      % POUNDING
   end

   if X21b20 - X31b30 - d2131 <= 0
     s2131 = 0; c2131 = 0;        % NO POUNDING BETWEEN BUILDINGS 2 & 3
   else
     s2131 = stiffness2131; c2131 = damping2131;      % POUNDING
   end


   % CHECK FOR SLIDING-----------------------------------------------------------------------------------------

   if mu1*(m11+Mb11)*g > abs( m11*Xoo11b11 + m11*Xoob11b10 + (m11+Mb11)*Xoog - f11 )
      a1 = 0;     % NO SLIDING OF BUILDING 1
   else
      a1 = 1      % SLIDING
   end

   if mu2*(m21+Mb21)*g > abs( m21*Xoo21b21 + m21*Xoob21b20 + (m21+Mb21)*Xoog - f21 )
      a2 = 0;     % NO SLIDING OF BUILDING 2
   else
      a2 = 1      % SLIDING
   end

   if mu3*(m31+Mb31)*g > abs( m31*Xoo31b31 + m31*Xoob31b30 + (m31+Mb31)*Xoog - f31 )
      a3 = 0;     % NO SLIDING OF BUILDING 3
   else
      a3 = 1      % SLIDING
   end


   % UPDATING THE INITIAL VALUES AND INTEGRATING-----------------------------------------------
```

```
initial = [result(j,3) ; result(j,9) ; result(j,15) ; result(j,6) ; result(j,12) ; result(j,18) ;result(j,4) ; result(j,10) ; result(j,16) ; result(j,7) ; result(j,13) ; result(j,19)];
parameters = [m11 m21 m31 c11 c21 c31 k11 k21 k31 c1121 c2131 s1121 s2131 Mb11 Mb21 Mb31 Ff1 Ff2 Ff3 a1 a2 a3 d1121 d2131];

options = odeset('MaxStep',0.01,'InitialStep',0.01,'AbsTol',1e-3);

[t,y] = ode45(@solver,[t1 t2],initial,options,parameters);


% CALCULATING THE NEXT TIME STEP VALUES AND SAVING IN 'result'-------------------------

n = length(y(:,1));  % GET THE LAST ROW OF y

time = t(n);
Xoog = f(time);
f11 = wind_force(time);
f21 = wind_force(time);
f31 = wind_force(time);

Xo11b11 = y(n,1);
X11b11 = y(n,7);
Xb11b10 = y(n,10);

Xo21b21 = y(n,2);
X21b21 = y(n,8);
Xb21b20 = y(n,11);

Xo31b31 = y(n,3);
X31b31 = y(n,9);
Xb31b30 = y(n,12);

if a1 == 0
   Xob11b10 = 0;
else
   Xob11b10 = y(n,4);
end

if a2 == 0
   Xob21b20 = 0;
else
   Xob21b20 = y(n,5);
end

if a3 == 0
   Xob31b30 = 0;
else
   Xob31b30 = y(n,6);
end

mass1 = [m11 0 0 0 0 0; 0 m21  0 0 0 0; 0 0 m31 0 0 0; a1*m11 0 0 m11+Mb11 0 0; 0 a2*m21 0 0 m21+Mb21 0; 0 0 a3*m31 0 0 m31+Mb31];
mass2 = [0 0 0 m11 0 0; 0 0 0 0 m21 0; 0 0 0 0 0 m31; 0 0 0 0 0 0; 0 0 0 0 0 0; 0 0 0 0 0 0];
```

```
mat3 = [c11 0 0 0 0 0; 0 c21 0 0 0 0; 0 0 c31 0 0 0;0 0 0 0 0 0;0 0 0 0 0 0;0 0 0 0 0 0];
mat4 = [c1121 -c1121 0 0 0 0; -c1121 c1121+c2131 -c2131 0 0 0; 0 -c2131 c2131 0 0 0; 0 0 0 0 0 0; 0 0 0 0 0 0;0 0 0 0 0 0];

mat5 = [0 0 0 k11 0 0;0 0 0 0 k21 0;0 0 0 0 0 k31;0 0 0 0 0 0;0 0 0 0 0 0;0 0 0 0 0 0];
mat6 = [0 0 0 s1121 -s1121 0;0 0 0 -s1121 s1121+s2131 -s2131;0 0 0 0 -s2131 s2131;0 0 0 0 0 0;0 0 0 0 0 0;0 0 0 0 0 0];

mat7 = [-s1121*d1121 ; s1121*d1121-s2131*d2131 ; s2131*d2131 ; 0; 0; 0];
%mat8 = [0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 1 0 0 0 0 0 0 0 0 0 0 0; 0 1 0 0 0 0 0 0 0 0 0 0; 0 0 1 0 0 0 0 0 0 0 0 0; 0 0 0 a1 0 0 0 0 0 0 0 0; 0 0 0 0 a2 0 0 0 0 0 0 0; 0 0 0 0 0 a3 0 0 0 0 0 0];

mat9 = [m11 0 0 0 0 0;0 m21 0 0 0 0;0 0 m31 0 0 0;0 0 0 0 0 0;0 0 0 0 0 0;0 0 0 0 0 0];

mat10 = [0 0 0 c1121 -c1121 0; 0 0 0 -c1121 c1121+c2131 -c2131;0 0 0 0 -c2131 c2131;0 0 0 0 0 0;0 0 0 0 0 0;0 0 0 0 0 0];

mat11 = [0 0 0 s1121 -s1121 0;0 0 0 -s1121 s1121+s2131 -s2131;0 0 0 0 -s2131 s2131;0 0 0 0 0 0;0 0 0 0 0 0;0 0 0 0 0 0];

Force = [f11;f21;f31; a1*f11+a1*(-sign(Xob11b10)*Ff1)-a1*(m11+Mb11)*Xoog ; a2*f21+a2*(-sign(Xob21b20)*Ff2)-a2*(m21+Mb21)*Xoog ; a3*f31+a3*(-sign(Xob31b30)*Ff3)-a3*(m31+Mb31)*Xoog];
E_quake = [Xoog;Xoog;Xoog;0;0;0];

matrice = inv(mass1+mass2)*(Force - mat9*E_quake - mat10*[0;0;0;Xob11b10;Xob21b20;Xob31b30] - mat11*[0;0;0;Xb11b10;Xb21b20;Xb31b30] - (mat3+mat4)*[Xo11b11;Xo21b21;Xo31b31;0;0;0] - (mat5+mat6)*[0;0;0;X11b11;X21b21;X31b31] - mat7);

Xoo11b11 = matrice(1);
Xoo21b21 = matrice(2);
Xoo31b31 = matrice(3);
Xoob11b10 = matrice(4);
Xoob21b20 = matrice(5);
Xoob31b30 = matrice(6);

t1 = t2;

j = j + 1


result(j,:) = [time Xoo11b11 Xo11b11 X11b11 Xoob11b10 Xob11b10 Xb11b10 Xoo21b21 Xo21b21 X21b21 Xoob21b20 Xob21b20 Xb21b20 Xoo31b31 Xo31b31 X31b31 Xoob31b30 Xob31b30 Xb31b30];

% PREDICTING THE IMPACT--------------------------------------------------------------------------------

X11b10 = X11b11 + Xb11b10;
X21b20 = X21b21 + Xb21b20;
X31b30 = X31b31 + Xb31b30;
```

```
    Xo11b10 = Xo11b11 + Xob11b10;
    Xo21b20 = Xo21b21 + Xob21b20;
    Xo31b30 = Xo31b31 + Xob31b30;

    if X11b10 - X21b20 - d1121 > 0
      impact_time1121(j12,1) = time;         % IMPACT BETWEEN BUILDINGS 1 & 2
      impact_time1121(j12,2) = X11b10;
      impact_time1121(j12,3) = X21b20;
      impact_time1121(j12,4) = Xo11b10;
      impact_time1121(j12,5) = Xo21b20;
      j12 = j12 + 1;
    end

    if X21b20 - X31b30 - d2131 > 0
      impact_time2131(j23,1) = time;         % IMPACT BETWEEN BUILDINGS 2 & 3
      impact_time2131(j23,2) = X21b20;
      impact_time2131(j23,3) = X31b30;
      impact_time2131(j23,4) = Xo21b20;
      impact_time2131(j23,5) = Xo31b30;
      j23 = j23 + 1;
    end

    %-----------------------------------------------------------------------------------------------------------------------

end

% SAVING THE MATRIX 'RESULT'-------------------------------------------------------------------------------

result;

save result

% PLOTS OF THE DISPLACEMENTS------------------------------------------------------------------------------

subplot(3,1,1), plot(result(:,1),result(:,4)+result(:,7),impact_time1121(:,1),impact_time1121(:,2),'*');
% grid on
title('X_1_1_,_B_1_0')
ylabel('Displacement (m)')
axis([0 50 -0.2 0.2])
hold on

subplot(3,1,2),
plot(result(:,1),result(:,10)+result(:,13),impact_time1121(:,1),impact_time1121(:,3),'*',impact_time2131(:,
1),impact_time2131(:,2),'+');
% grid on
title('X_2_1_,_B_2_0')
ylabel('Displacement (m)')
axis([0 50 -0.2 0.2])
hold on

subplot(3,1,3), plot(result(:,1),result(:,16)+result(:,19),impact_time2131(:,1),impact_time2131(:,3),'+');
% grid on
title('X_3_1_,_B_3_0')
```

```
xlabel('Time (sec)')
ylabel('Displacement (m)')
axis([0 50 -0.2 0.2])
hold on

% THIS IS THE END OF THE 'MAIN' FUNCTION-------------------------------------------------------------



% ‘SOLVER’ FUNCTION
% THIS FUNCTION IS CALLED BY THE 'main' FUNCTION

function dy = solver(t,y,parameters)

m11 = parameters(1);
m21 = parameters(2);
m31 = parameters(3);
c11 = parameters(4);
c21 = parameters(5);
c31 = parameters(6);
k11 = parameters(7);
k21 = parameters(8);
k31 = parameters(9);
c1121 = parameters(10);
c2131 = parameters(11);
s1121 = parameters(12);
s2131 = parameters(13);
Mb11 = parameters(14);
Mb21 = parameters(15);
Mb31 = parameters(16);
Ff1 = parameters(17);
Ff2 = parameters(18);
Ff3 = parameters(19);
a1 = parameters(20);
a2 = parameters(21);
a3 = parameters(22);
d1121 = parameters(23);
d2131 = parameters(24);

ag =  f(t);
f11 = wind_force(t);
f21 = wind_force(t);
f31 = wind_force(t);

dy = zeros(12,1);

M1 = [m11 0 0 0 0 0 0 0 0 0 0 0; 0 m21  0 0 0 0 0 0 0 0 0 0; 0 0 m31 0 0 0 0 0 0 0 0 0; a1*m11 0 0
m11+Mb11 0 0 0 0 0 0 0 0; 0 a2*m21 0 0 m21+Mb21 0 0 0 0 0 0 0; 0 0 a3*m31 0 0 m31+Mb31 0 0 0 0 0
0; 0 0 0 0 0 0 1 0 0 0 0 0; 0 0 0 0 0 0 0 1 0 0 0 0; 0 0 0 0 0 0 0 0 1 0 0 0; 0 0 0 0 0 0 0 0 0 1 0 0; 0 0 0 0 0 0
0 0 0 0 1 0; 0 0 0 0 0 0 0 0 0 0 0 1];
```

```
M2 = [0 0 0 m11 0 0 0 0 0 0 0 0; 0 0 0 0 m21 0 0 0 0 0 0 0; 0 0 0 0 0 m31 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0
0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0
0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0];
M3 = [m11 0 0 0 0 0 0 0 0 0 0 0; 0 m21 0 0 0 0 0 0 0 0 0 0; 0 0 m31 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0
0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0
0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0];

C1 = [c11 0 0 0 0 0 0 0 0 0 0 0; 0 c21 0 0 0 0 0 0 0 0 0 0; 0 0 c31 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0;
0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0
0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0];
C2 = [c1121 -c1121 0 0 0 0 0 0 0 0 0 0; -c1121 c1121+c2131 -c2131 0 0 0 0 0 0 0 0 0; 0 -c2131 c2131 0 0
0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0;
0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0
0 0 0 0 0];
C3 = [0 0 0 c1121 -c1121 0 0 0 0 0 0 0; 0 0 0 -c1121 c1121+c2131 -c2131 0 0 0 0 0 0; 0 0 0 0 -c2131
c2131 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0
0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0
0 0 0 0 0 0 0];

K1 = [0 0 0 0 0 0 k11 0 0 0 0 0; 0 0 0 0 0 0 0 k21 0 0 0 0; 0 0 0 0 0 0 0 0 k31 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0;
0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0
0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0];
K2 = [0 0 0 0 0 0 s1121 -s1121 0 0 0 0; 0 0 0 0 0 0 -s1121 s1121+s2131 -s2131 0 0 0; 0 0 0 0 0 0 0 -s2131
s2131 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0
0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0
0 0 0 0];
K3 = [0 0 0 0 0 0 0 0 0 s1121 -s1121 0; 0 0 0 0 0 0 0 0 0 -s1121 s1121+s2131 -s2131; 0 0 0 0 0 0 0 0 0 0 -
s2131 s2131; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0;
0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0
0 0 0 0 0];

D1 = [-s1121*d1121;s1121*d1121-s2131*d2131;s2131*d2131; 0; 0; 0; 0; 0; 0; 0; 0; 0];
D2 = [0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0
0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 1 0 0 0 0 0 0 0 0 0 0 0; 0 1 0 0 0 0 0 0 0 0 0 0; 0 0 1 0 0 0 0 0 0 0 0
0; 0 0 0 a1 0 0 0 0 0 0 0 0; 0 0 0 0 a2 0 0 0 0 0 0 0; 0 0 0 0 0 a3 0 0 0 0 0 0];


dy = inv(M1+M2)*([f11;f21;f31; a1*f11+a1*(-sign(y(4))*Ff1)-a1*(m11+Mb11)*ag ; a2*f21+a2*(-
sign(y(5))*Ff2)-a2*(m21+Mb21)*ag ; a3*f31+a3*(-sign(y(6))*Ff3)-a3*(m31+Mb31)*ag ;0;0;0;0;0;0] -
M3*[ag;ag;ag;0;0;0;0;0;0;0;0;0] - (C1+C2+C3+K1+K2+K3-D2)*y(1:12,1) - D1);

% THIS IS THE END OF THE 'SOLVER' FUNCTION-----------------------------------------------------------



% 'F' FUNCTION
% THIS FUNCTION OUTPUTS THE GROUND ACCELERATION OF EARTHQUAKE AT TIME t

function accl = f(t)

% HARMONIC FUNCTION EARTHQUAKE
% accl = 10*sin(10*t);
```

```
% EL CENTRO EARTHQUAKE
g = 9.814;                  % Accleration due to gravity, m/sec2.
time = 0:0.02:49.98;
accl = g*spline(time,elcentro_data,t);      % Ground acceleration

% MEXICO CITY EARTHQUAKE
% g = 9.814;                % Accleration due to gravity, m/sec2.
% time = 0:0.02:180.08;
% accl = g*spline(time,mexico_city,t);    % Ground acceleration

% LOMA PRIETA EARTHQUAKE
% g = 9.814;                % Accleration due to gravity, m/sec2.
% time = 0:0.02:40;
% accl = g*spline(time,loma,t);             % Ground acceleration

% NORTHRIDGE EARTHQUAKE
% g = 9.814;                % Accleration due to gravity, m/sec2.
% time = 0:0.02:24.04;
% accl = g*spline(time,northridge_ca,t); % Ground acceleration

% THIS IS THE END OF 'F' FUNCTION--------------------------------------------------------------------------



% ‘WIND_FORCE’ FUNCTION
% THIS FUNCTION OUTPUTS THE WIND FORCE

function force = wind_force(t)

force = 0;

% THIS IS THE END OF ' WIND_FORCE ' FUNCTION-------------------------------------------------------------



% ‘ELCENTRO_DATA’ FUNCTION
% THIS FUNCTION CONTAINS EL CENTRO EARTHQUAKE DATA.
%
% THE DATA IS IN A NON DIMENSIONAL FORM OBTAINED BY DIVIDING
% BY ACCELERATION DUE TO GRAVITY.
%
% Total readings  First reading    Delta t
%     2500            1             .02

function y = elcentro_data

y= [

EL CENTRO EARTHQUAKE DATA
```

```
];

% THIS IS THE END OF ' ELCENTRO_DATA ' FUNCTION--------------------------------------------------



% ‘MEXICO_CITY’ FUNCTION
% THIS FUNCTION CONTAINS MEXICO CITY EARTHQUAKE DATA.
%
% THE DATA IS IN A NON DIMENSIONAL FORM OBTAINED BY DIVIDING
% BY ACCELERATION DUE TO GRAVITY.
%

function y = mexico_city

%9004  1  .02

y = [0

MEXICO CITY EARTHQUAKE DATA

];

% THIS IS THE END OF ' MEXICO_CITY ' FUNCTION-------------------------------------------------------



% ‘LOMA’ FUNCTION
% THIS FUNCTION CONTAINS LOMA PRIETA EARTHQUAKE DATA.
%
% THE DATA IS IN A NON DIMENSIONAL FORM OBTAINED BY DIVIDING
% BY ACCELERATION DUE TO GRAVITY.
%

function y = loma

% 2000  1  .02

y = [0

LOMA PRIETA EARTHQUAKE DATA

];

% THIS IS THE END OF 'LOMA' FUNCTION------------------------------------------------------------------



% ‘NORTHRIDGE_CA’ FUNCTION
% THIS FUNCTION CONTAINS NORTHRIDGE EARTHQUAKE DATA.
```

```
%
% THE DATA IS IN A NON DIMENSIONAL FORM OBTAINED BY DIVIDING
% BY ACCELERATION DUE TO GRAVITY.

function y = northridge_ca

% 1202  1  .02

y = [0

NORTHRIDGE EARTHQUAKE DATA

];

% THIS IS THE END OF ' NORTHRIDGE_CA ' FUNCTION---------------------------------------------------
```

APPENDIX 4

COMPUTER PROGRAMS IN MATLAB TO OBTAIN RESPONSE OF TWO ADJACENT TWO DEGREE OF FREEDOM BASE ISOLATED BUILDINGS

```
% ‘MAIN’ FUNCTION
% THIS PROGRAM CALCULATES THE RESPONSE OF TWO ADJACENT 2-DOF BASE
% ISOLATED SYSTEM. THE OUTPUT RESPONSE IS STORED IN THE VECTOR 'RESULT'
% WHICH HAVE NINTEEN COLUMNS. THE FIRST COLUMN IS THE TIME VECTOR AND
% OTHER EIGHTEEN COLUMNS CONTAINS THE ACCELERATION, VELOCITY AND
% DISPLACEMENT RESPONSES
%
% INPUT COMMAND IS 'main(h,to)'
% WHERE h = TIME STEP AND to = MAXIMUM TIME OF SIMULATION
%
% OTHER PROGRAMS WHICH IT CALLS ARE 'solver', 'f', 'wind_force', 'elcentro_data',
% 'mexico_city', 'loma', 'northridge_ca'
%
%
% INPUT PARAMETERS LIKE THE STRUCTURAL MASS, STIFFNESS AND DAMPING, IMPACT
% STIFFNESS AND DASHPOTS, SPACING BETWEEN THE BUILDINGS, MASS OF THE BASE
% OF STRUCTURE AND BASE ISOLATION FRICTION COEFFICIENT SHOULD BE CHANGED
% WITHIN THE PROGRAM.
%
% ALL INPUTS ARE IN SI UNITS
%
% IN THIS PROGRAM 'AbsTol' HAS BEEN REDUCED TO 1e-3
%
% WRITTEN BY
%
% VIVEK KUMAR AGARWAL, SPRING 2004

function main(h,to)

% INPUT PARAMETERS----------------------------------------------------------------------------------------------

% ALL PARAMETERS IN SI UNITS

m11 = 26268;       % STRUCTURAL MASS
m12 = 17512;
m21 = 52536;
m22 = 35024;

c11 = 23634.2;     % STRUCTURAL DAMPING
c12 = 16881.57;
c21 = 33343;
c22 = 23816;

k11 = 24516800;  % STRUCTURAL STIFFNESS
k12 = 17512000;
k21 = 24516800;
k22 = 17512000;

damping1121 = 430384.4;      % IMPACT DASHPOT BETWEEN FIRST STORY
damping1222 = 286923;
stiffness1121 = 56993585;     % IMPACT STIFFNESS BETWEEN FIRST STORY
stiffness1222 = 37995723.36;
d1121 = 150.01;                     % SPACING BETWEEN THE BUILDINGS
```

```
d1222 = 150.01;

Mb11 = 13134;    % MASS OF THE BASE OF STRUCTURE
Mb21 = 26268;
mu1 = 150.10;    % FRICTION COEFFICIENT IN BASE ISOLATION
mu2 = 100.10;

g = 9.814;       % ACCELEATION DUE TO GRAVITY

Ff1 = mu1*(m11+m12+Mb11)*g;
Ff2 = mu2*(m21+m22+Mb21)*g;

% INITIAL CONDITIONS--------------------------------------------------------------------------------------------
j =1;
t1 = 0;
time = 0;
Xoog = 0;
f11 = 0;
f12 = 0;
f21 = 0;
f22 = 0;

Xoo11b11 = 0;    % INITIAL CONDITIONS FOR BUILDING 1
Xo11b11 = 0;
X11b11 = 0;
Xoo12b11 = 0;
Xo12b11 = 0;
X12b11 = 0;
Xoob11b10 = 0;
Xob11b10 = 0;
Xb11b10 = 0;

Xoo21b21 = 0;    % INITIAL CONDITIONS FOR BUILDING 2
Xo21b21 = 0;
X21b21 = 0;
Xoo22b21 = 0;
Xo22b21 = 0;
X22b21 = 0;
Xoob21b20 = 0;
Xob21b20 = 0;
Xb21b20 = 0;


X11b10 = 0;
X12b10 = 0;
X21b20 = 0;
X22b20 = 0;

a1 = 0;
a2 = 0;
```

```
result(j,:) = [time  Xoo11b11  Xo11b11  X11b11  Xoo12b11  Xo12b11  X12b11  Xoob11b10  Xob11b10
Xb11b10   Xoo21b21   Xo21b21   X21b21   Xoo22b21   Xo22b21   X22b21  Xoob21b20   Xob21b20
Xb21b20];
% COLUMN NUMBER
%           1      2      3      4      5      6      7      8      9      10      11      12      13      14      15
% 16      17      18      19

% INITIALIZE j12, impact_time1121(j12,:), j23, impact_time2131(j23,:)
j1121 = 1;
impact_time1121(j1121,:) = [0 0 0];
j1222 = 1;
impact_time1222(j1222,:) = [0 0 0];

% 'FOR' LOOP TO COMPUTE FOR EACH TIME STEP--------------------------------------------------------

for t2 = h:h:to

   % CHECK FOR POUNDING---------------------------------------------------------------------------------

   if X11b10 - X21b20 - d1121 <= 0
     s1121 = 0; c1121 = 0;    % NO POUNDING OF FIRST STORIES
   else
     s1121 = stiffness1121; c1121 = damping1121;  % POUNDING
   end

   if X12b10 - X22b20 - d1222 <= 0
     s1222 = 0; c1222 = 0;    % NO POUNDING OF SECOND STORIES
   else
     s1222 = stiffness1222; c1222 = damping1222;  % POUNDING
   end


   % CHECK FOR SLIDING---------------------------------------------------------------------------------

   if mu1*(m11+m12+Mb11)*g > abs( m11*Xoo11b11 + m12*Xoo12b11 + (m11+m12)*Xoob11b10 +
(m11+m12+Mb11)*Xoog - f11 - f12 )
      a1 = 0;     % No SLIDING OF BUILDING 1
   else
      a1 = 1      % SLIDING
   end

   if mu2*(m21+m22+Mb21)*g > abs( m21*Xoo21b21 + m22*Xoo22b21 + (m21+m22)*Xoob21b20 +
(m21+m22+Mb21)*Xoog - f21 - f22 )
      a2 = 0;     % No SLIDING OF BUILDING 2
   else
      a2 = 1      % SLIDING
   end


   % UPDATING THE INITIAL VALUES AND INTEGRATING----------------------------------------------

   initial = [result(j,3) ; result(j,6) ; result(j,12) ; result(j,15) ; result(j,9) ; result(j,18) ; result(j,4) ;
result(j,7) ; result(j,13) ; result(j,16) ; result(j,10) ; result(j,19) ];
```

```
parameters = [m11 m12 m21 m22 c11 c12 c21 c22 k11 k12 k21 k22 c1121 c1222 s1121 s1222 d1121 d1222 Mb11 Mb21 Ff1 Ff2 a1 a2];

options = odeset('MaxStep',0.01,'InitialStep',0.01,'AbsTol',1e-3);

[t,y] = ode45(@solver,[t1 t2],initial,options,parameters);


% CALCULATING THE VALUES AT NEXT TIME STEP AND STORING IN 'RESULT'--------------

n = length(y(:,1));       % GET THE LAST ROW OF Y

time = t(n);
Xoog = f(time);
f11 = wind_force(time);
f12 = wind_force(time);
f21 = wind_force(time);
f22 = wind_force(time);

Xo11b11 = y(n,1);
X11b11 = y(n,7);
Xo12b11 = y(n,2);
X12b11 = y(n,8);
Xb11b10 = y(n,11);

Xo21b21 = y(n,3);
X21b21 = y(n,9);
Xo22b21 = y(n,4);
X22b21 = y(n,10);
Xb21b20 = y(n,12);

if a1 == 0
   Xob11b10 = 0;
else
   Xob11b10 = y(n,5);
end

if a2 == 0
   Xob21b20 = 0;
else
   Xob21b20 = y(n,6);
end

mass1 = [m11 0 0 0 0 0; 0 m12  0 0 0 0; 0 0 m21 0 0 0; 0 0 0 m22 0 0; a1*m11 a1*m12 0 0 m11+m12+Mb11 0; 0 0 a2*m21 a2*m22 0 m21+m22+Mb21];
mass2 = [0 0 0 0 m11 0; 0 0 0 0 m12 0; 0 0 0 0 0 m21; 0 0 0 0 0 m22; 0 0 0 0 0 0; 0 0 0 0 0 0];

mat3 = [c11+c12 -c12 0 0 0 0; -c12 c12 0 0 0 0; 0 0 c21+c22 -c22 0 0; 0 0 -c22 c22 0 0; 0 0 0 0 0 0; 0 0 0 0 0 0];
mat4 = [c1121 0 -c1121 0 0 0; 0 c1222 0 -c1222 0 0; -c1121 0 c1121 0 0 0; 0 -c1222 0 c1222 0 0; 0 0 0 0 0 0; 0 0 0 0 0 0];
```

```
mat5 = [k11+k12 -k12 0 0 0 0; -k12 k12 0 0 0 0; 0 0 k21+k22 -k22 0 0; 0 0 -k22 k22 0 0; 0 0 0 0 0 0; 0 0 0 0 0 0];
mat6 = [s1121 0 -s1121 0 0 0; 0 s1222 0 -s1222 0 0; -s1121 0 s1121 0 0 0; 0 -s1222 0 s1222 0 0; 0 0 0 0 0 0; 0 0 0 0 0 0];

mat7 = [-s1121*d1121;-s1222*d1222;s1121*d1121; s1222*d1222; 0; 0];

mat9 = [m11 0 0 0 0 0; 0 m12 0 0 0 0; 0 0 m21 0 0 0; 0 0 0 m22 0 0; 0 0 0 0 0 0; 0 0 0 0 0 0];

mat10 = [0 0 0 0 c1121 -c1121 ; 0 0 0 0 c1222 -c1222 ; 0 0 0 0 -c1121 c1121 ; 0 0 0 0 -c1222 c1222 ; 0 0 0 0 0 0 ; 0 0 0 0 0 0];

mat11 = [0 0 0 0 s1121 -s1121 ; 0 0 0 0 s1222 -s1222 ; 0 0 0 0 -s1121 s1121 ; 0 0 0 0 -s1222 s1222 ; 0 0 0 0 0 0 ; 0 0 0 0 0 0];

Force = [f11;f12;f21;f22; a1*(f11+f12)+a1*(-sign(Xob11b10)*Ff1)-a1*(m11+m12+Mb11)*Xoog ; a2*(f21+f22)+a2*(-sign(Xob21b20)*Ff2)-a2*(m21+m22+Mb21)*Xoog];
E_quake = [Xoog;Xoog;Xoog;Xoog;0;0];

matrice = inv(mass1+mass2)*(Force - mat9*E_quake - mat10*[0;0;0;0;Xob11b10;Xob21b20]-mat11*[0;0;0;0;Xb11b10;Xb21b20]-(mat3+mat4)*[Xo11b11;Xo12b11;Xo21b21;Xo22b21;0;0]-(mat5+mat6)*[X11b11;X12b11;X21b21;X22b21;0;0]-mat7);

Xoo11b11 = matrice(1);
Xoo12b11 = matrice(2);
Xoo21b21 = matrice(3);
Xoo22b21 = matrice(4);
Xoob11b10 = matrice(5);
Xoob21b20 = matrice(6);

t1 = t2;

j = j + 1

result(j,:) = [time Xoo11b11 Xo11b11 X11b11 Xoo12b11 Xo12b11 X12b11 Xoob11b10 Xob11b10 Xb11b10 Xoo21b21 Xo21b21 X21b21 Xoo22b21 Xo22b21 X22b21 Xoob21b20 Xob21b20 Xb21b20];


% TO KEEP TRACK OF THE IMPACTS-------------------------------------------------------------------------

X11b10 = X11b11 + Xb11b10;
X12b10 = X12b11 + Xb11b10;
X21b20 = X21b21 + Xb21b20;
X22b20 = X22b21 + Xb21b20;

Xo11b10 = Xo11b11 + Xob11b10;
Xo12b10 = Xo12b11 + Xob11b10;
Xo21b20 = Xo21b21 + Xob21b20;
Xo22b20 = Xo22b21 + Xob21b20;

if X11b10 - X21b20 - d1121 > 0       % IMPACT BETWEEN FIRST STORIES
  impact_time1121(j1121,1) = time;
```

```
    impact_time1121(j1121,2) = X11b10;
    impact_time1121(j1121,3) = X21b20;
    impact_time1121(j1121,4) = Xo11b10;
    impact_time1121(j1121,5) = Xo21b20;
    j1121 = j1121 + 1;
  end

  if X12b10 - X22b20 - d1222 > 0      % IMPACT BETWEEN SECOND STORIES
    impact_time1222(j1222,1) = time;
    impact_time1222(j1222,2) = X12b10;
    impact_time1222(j1222,3) = X22b20;
    impact_time1222(j1222,4) = Xo12b10;
    impact_time1222(j1222,5) = Xo22b20;
    j1222 = j1222 + 1;
  end

  %-------------------------------------------------------------------------------------------------------------------------

end

% SAVE THE RESULT-------------------------------------------------------------------------------------------------

result;

save result

% PLOT THE TOTAL DISPLACEMENTS---------------------------------------------------------------------------

figure(1)
subplot(2,1,1), plot(result(:,1),result(:,7)+result(:,10),impact_time1222(:,1),impact_time1222(:,2),'*');
%grid on
title('X_1_2_,_B_1_0')
xlabel('Time (sec)')
ylabel('Displacement (m)')
axis([0 50 -0.05 0.05])
set(gca,'YTick',-0.05:0.01:0.05)
set(gca,'XTick',0:5:50)
%legend('mass','\Omega = 10 rad/sec^2',1)
hold on
% legend('\Omega = 2 rad/sec^2','\Omega = 10 rad/sec^2',1)

subplot(2,1,2), plot(result(:,1),result(:,4)+result(:,10),impact_time1121(:,1),impact_time1121(:,2),'*');
%grid on
title('X_1_1_,_B_1_0')
xlabel('Time (sec)')
ylabel('Displacement (m)')
axis([0 50 -0.05 0.05])
set(gca,'YTick',-0.05:0.01:0.05)
set(gca,'XTick',0:5:50)
hold on
% legend('\Omega = 2 rad/sec^2','\Omega = 10 rad/sec^2',1)

figure(2)
```

```
subplot(2,1,1), plot(result(:,1),result(:,16)+result(:,19),impact_time1222(:,1),impact_time1222(:,3),'*');
%grid on
title('X_2_2_,_B_2_0')
xlabel('Time (sec)')
ylabel('Displacement (m)')
axis([0 50 -0.05 0.05])
set(gca,'YTick',-0.05:0.01:0.05)
set(gca,'XTick',0:5:50)
hold on
% legend('\Omega = 2 rad/sec^2','\Omega = 10 rad/sec^2',1)

subplot(2,1,2), plot(result(:,1),result(:,13)+result(:,19),impact_time1121(:,1),impact_time1121(:,3),'*');
%grid on
title('X_2_1_,_B_2_0')
xlabel('Time (sec)')
ylabel('Displacement (m)')
axis([0 50 -0.05 0.05])
set(gca,'YTick',-0.05:0.01:0.05)
set(gca,'XTick',0:5:50)
hold on

% THIS IS THE END OF 'MAIN' FUNCTION-------------------------------------------------------------------



% ‘SOLVER’ FUNCTION
% THIS FUNCTION IS CALLED BY 'main' FUNCTION

function dy = solver(t,y,parameters)

m11 = parameters(1);
m12 = parameters(2);
m21 = parameters(3);
m22 = parameters(4);
c11 = parameters(5);
c12 = parameters(6);
c21 = parameters(7);
c22 = parameters(8);
k11 = parameters(9);
k12 = parameters(10);
k21 = parameters(11);
k22 = parameters(12);
c1121 = parameters(13);
c1222 = parameters(14);
s1121 = parameters(15);
s1222 = parameters(16);
d1121 = parameters(17);
d1222 = parameters(18);
Mb11 = parameters(19);
Mb21 = parameters(20);
Ff1 = parameters(21);
Ff2 = parameters(22);
```

```
a1 = parameters(23);
a2 = parameters(24);


ag =  f(t);
f11 = wind_force(t);
f12 = wind_force(t);
f21 = wind_force(t);
f22 = wind_force(t);

dy = zeros(12,1);

M1 = [m11 0 0 0 0 0 0 0 0 0 0 0; 0 m12  0 0 0 0 0 0 0 0 0 0; 0 0 m21 0 0 0 0 0 0 0 0 0; 0 0 0 m22 0 0 0 0 0 0 0 0; a1*m11 a1*m12 0 0 m11+m12+Mb11 0 0 0 0 0 0 0; 0 0 a2*m21 a2*m22 0 m21+m22+Mb21 0 0 0 0 0 0; 0 0 0 0 0 0 1 0 0 0 0 0; 0 0 0 0 0 0 0 1 0 0 0 0; 0 0 0 0 0 0 0 0 1 0 0 0; 0 0 0 0 0 0 0 0 0 1 0 0; 0 0 0 0 0 0 0 0 0 0 1 0; 0 0 0 0 0 0 0 0 0 0 0 1];
M2 = [0 0 0 0 m11 0 0 0 0 0 0 0; 0 0 0 0 m12 0 0 0 0 0 0 0; 0 0 0 0 0 m21 0 0 0 0 0 0; 0 0 0 0 0 m22 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0];
M3 = [m11 0 0 0 0 0 0 0 0 0 0 0; 0 m12 0 0 0 0 0 0 0 0 0 0; 0 0 m21 0 0 0 0 0 0 0 0 0; 0 0 0 m22 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0];

C1 = [c11+c12 -c12 0 0 0 0 0 0 0 0 0 0; -c12 c12 0 0 0 0 0 0 0 0 0 0; 0 0 c21+c22 -c22 0 0 0 0 0 0 0 0; 0 0 -c22 c22 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0];
C2 = [c1121 0 -c1121 0 0 0 0 0 0 0 0 0; 0 c1222 0 -c1222 0 0 0 0 0 0 0 0; -c1121 0 c1121 0 0 0 0 0 0 0 0 0; 0 -c1222 0 c1222 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0];
C3 = [0 0 0 0 c1121 -c1121 0 0 0 0 0 0; 0 0 0 0 c1222 -c1222 0 0 0 0 0 0; 0 0 0 0 -c1121 c1121 0 0 0 0 0 0; 0 0 0 0 -c1222 c1222 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0];

K1 = [0 0 0 0 0 0 k11+k12 -k12 0 0 0 0; 0 0 0 0 0 0 -k12 k12 0 0 0 0; 0 0 0 0 0 0 0 0 k21+k22 -k22 0 0; 0 0 0 0 0 0 0 0 -k22 k22 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0];
K2 = [0 0 0 0 0 0 s1121 0 -s1121 0 0 0; 0 0 0 0 0 0 0 s1222 0 -s1222 0 0; 0 0 0 0 0 0 -s1121 0 s1121 0 0 0; 0 0 0 0 0 0 0 -s1222 0 s1222 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0];
K3 = [0 0 0 0 0 0 0 0 0 0 s1121 -s1121; 0 0 0 0 0 0 0 0 0 0 s1222 -s1222; 0 0 0 0 0 0 0 0 0 0 -s1121 s1121; 0 0 0 0 0 0 0 0 0 0 -s1222 s1222; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0];

D1 = [-s1121*d1121;-s1222*d1222;s1121*d1121; s1222*d1222; 0; 0; 0; 0; 0; 0; 0; 0];
D2 = [0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0; 1 0 0 0 0 0 0 0 0 0 0 0; 0 1 0 0 0 0 0 0 0 0 0 0; 0 0 1 0 0 0 0 0 0 0 0 0; 0 0 0 1 0 0 0 0 0 0 0 0; 0 0 0 0 a1 0 0 0 0 0 0 0; 0 0 0 0 0 a2 0 0 0 0 0 0];
```

```
dy = inv(M1+M2)*([f11;f12;f21;f22; a1*(f11+f12)+a1*(-sign(y(5))*Ff1)-a1*(m11+m12+Mb11)*ag ;
a2*(f21+f22)+a2*(-sign(y(6))*Ff2)-a2*(m21+m22+Mb21)*ag ;0;0;0;0;0;0] -
M3*[ag;ag;ag;ag;0;0;0;0;0;0;0;0]-(C1+C2+C3)*y(1:12,1)-(K1+K2+K3)*y(1:12,1)-D1+D2*y(1:12,1));

% THIS IS THE END OF 'SOLVER' FUNCTION-----------------------------------------------------------------



% ‘F’ FUNCTION
% THIS FUNCTION OUTPUTS THE GROUND ACCELERATION OF EARTHQUAKE AT TIME t

function accl = f(t)

% HARMONIC FUNCTION EARTHQUAKE
% accl = 10*sin(10*t);

% EL CENTRO EARTHQUAKE
g = 9.814;                    % Accleration due to gravity, m/sec2.
time = 0:0.02:49.98;
accl = g*spline(time,elcentro_data,t);        % Ground acceleration

% MEXICO CITY EARTHQUAKE
% g = 9.814;                  % Accleration due to gravity, m/sec2.
% time = 0:0.02:180.08;
% accl = g*spline(time,mexico_city,t);      % Ground acceleration

% LOMA PRIETA EARTHQUAKE
% g = 9.814;                  % Accleration due to gravity, m/sec2.
% time = 0:0.02:40;
% accl = g*spline(time,loma,t);             % Ground acceleration

% NORTHRIDGE EARTHQUAKE
% g = 9.814;                  % Accleration due to gravity, m/sec2.
% time = 0:0.02:24.04;
% accl = g*spline(time,northridge_ca,t);   % Ground acceleration

% THIS IS THE END OF 'F' FUNCTION-------------------------------------------------------------------------



% ‘WIND_FORCE’ FUNCTION
% THIS FUNCTION OUTPUTS THE WIND FORCE

function force = wind_force(t)

force = 0;

% THIS IS THE END OF ‘WIND_FORCE’ FUNCTION-------------------------------------------------------
```

```
% 'ELCENTRO_DATA' FUNCTION
% THIS FUNCTION CONTAINS EL CENTRO EARTHQUAKE DATA.
%
% THE DATA IS IN A NON DIMENSIONAL FORM OBTAINED BY DIVIDING
% BY ACCELERATION DUE TO GRAVITY.
%
% Total readings  First reading    Delta t
%    2500            1              .02

function y = elcentro_data

y= [

EL CENTRO EARTHQUAKE DATA

];

% THIS IS THE END OF ' ELCENTRO_DATA' FUNCTION--------------------------------------------------



% 'MEXICO_CITY' FUNCTION
% THIS FUNCTION CONTAINS MEXICO CITY EARTHQUAKE DATA.
%
% THE DATA IS IN A NON DIMENSIONAL FORM OBTAINED BY DIVIDING
% BY ACCELERATION DUE TO GRAVITY.

function y = mexico_city

%9004  1  .02

y = [0

MEXICO CITY EARTHQUAKE DATA

];

% THIS IS THE END OF ' MEXICO_CITY' FUNCTION--------------------------------------------------------



% 'LOMA' FUNCTION
% THIS FUNCTION CONTAINS LOMA PRIETA EARTHQUAKE DATA.
%
% THE DATA IS IN A NON DIMENSIONAL FORM OBTAINED BY DIVIDING
% BY ACCELERATION DUE TO GRAVITY.

function y = loma

% 2000  1  .02
```

```
y = [0

LOMA PRIETA EARTHQUAKE DATA

];

% THIS IS THE END OF 'LOMA' FUNCTION--------------------------------------------------------------------



% ‘NORTHRIDGE_CA’ FUNCTION
% THIS FUNCTION CONTAINS NORTHRIDGE EARTHQUAKE DATA.
%
% THE DATA IS IN A NON DIMENSIONAL FORM OBTAINED BY DIVIDING
% BY ACCELERATION DUE TO GRAVITY.
%

function y = northridge_ca

% 1202  1  .02

y = [0

NORTHRIDGE EARTHQUAKE DATA

];

% THIS IS THE END OF ' NORTHRIDGE_CA' FUNCTION----------------------------------------------------
```

APPENDIX 5

COMPUTER PROGRAMS IN MATLAB TO OBTAIN THE RESPONSE OF SINGLE MULTI DEGREE OF FREEDOM SYSTEM USING MODAL ANALYSIS

```
% ‘MAIN’ FUNCTION
% THIS PROGRAM CALCULATES THE RESPONSE OF SINGLE MDOF FIXED BASE BUILDING.
% THE OUTPUT RESPONSE IS STORED IN THE VECTOR 'x' WHICH HAVE COLUMNS EQUAL
% TO NUMBER OF DEGREES OF FREEDOM. EACH COLUMN IS THE DISPLACEMENT
% RESPONSE OF LUMPED MASSES.
%
% INPUT COMMAND IS 'main(m,k,beta,to)'
% WHERE m = MASS MATRIX, k = STIFFNESS MATRIX, beta = CRITICAL DAMPING RATIO
% ASSUMING SAME DAMPING IN ALL MODES AND to = MAXIMUM TIME OF SIMULATION
%
% OTHER PROGRAMS WHICH IT CALLS ARE 'solver', 'f', 'displacement', 'elcentro_data'
%
% ALL INPUTS ARE IN SI UNITS
%
% WRITTEN BY
%
% VIVEK KUMAR AGARWAL, SPRING 2004


function main(m,k,beta,to)


% CALCULATION OF MODE SHAPES AND NATURAL FREQUENCIES
[phi,w] = eig(inv(m)*k);
W = sqrt(diag(w));
n = size(m);
dof = n(1);

% CONSTRUCTING THE DAMPING MATRIX
% C(1,1) = 2*BETA1*W(1);     % FOR DIFFERENT DAMPING IN DIFFERENT MODES
% C(2,2) = 2*BETA2*W(2);

C = zeros(dof,dof);                  % SAME DAMPING IN ALL MODES
for i = 1:1:dof
   C(i,i) = 2*beta*W(i);
end

% GENERALIZED MASS, STIFFNESS AND LOAD
M = phi'*m*phi;
K = phi'*k*phi;

I = ones(dof,1);
XX = phi'*m*I;

% CALCULATION OF MODE PARTICIPATION FACTOR
for j = 1:1:dof
   factor(j) = (XX(j))/(M(j,j));
end

% INTEGRATION FOR EACH MODE
for j = 1:1:dof
   options = odeset('MaxStep',0.01,'InitialStep',0.01);
   [t,y] = ode45(@solver,[0 to],[0 0],options,M,C,W,phi,factor,j);
```

```
    q(j,:) = (y(:,2))';
end

% CALCULATING BACK THE DISLACEMENT RESPONSE
displacement(t,q,phi)

% THIS IS THE END OF THE 'MAIN' FUNCTION------------------------------------------------------------



% 'SOLVER' FUNCTION
% THIS FUNCTION IS CALLED BY THE 'main' FUNCTION

function dy = solver(t,y,M,C,W,phi,factor,j)

dy = zeros(2,1);

F = -factor(j)*f(t);

dy(1) = F - C(j,j)*y(1) - W(j)*W(j)*y(2);

dy(2) = y(1);

% THIS IS THE END OF THE 'SOLVER' FUNCTION----------------------------------------------------------



% 'DISPLACEMENT' FUNCTION
% THIS FUNCTION IS CALLED BY 'main' FUNCTION AND CALCULATES THE
% RESPONSE OF THE STRUCTURE FROM GENERALIZED COORDINATES.

function x = displacement(t,q,phi)

x = (phi*q)';  % USING GENERALIZED COORDINATES TO GET BACK THE RESPONSE

save t;
save x;

% plot(t,x(:,1),t,x(:,2),'k')
% grid on;
% xlabel('Displacement of a Mass, x')
% ylabel('Time (sec)')

subplot(2,1,1), plot(t,x(:,2));
%grid on
title('X_1_2_,_B_1_0')
xlabel('Time (sec)')
ylabel('Displacement (m)')
axis([0 15 -0.05 0.05])
hold on
legend('\Omega = 2 rad/sec^2','\Omega = 10 rad/sec^2',1)
```

```
subplot(2,1,2), plot(t,x(:,1));
%grid on
title('X_1_1_,_B_1_0')
xlabel('Time (sec)')
ylabel('Displacement (m)')
axis([0 15 -0.05 0.05])
hold on
legend('\Omega = 2 rad/sec^2','\Omega = 10 rad/sec^2',1)

% THIS IS THE END OF THE 'DISPLACEMENT' FUNCTION------------------------------------------------


% ‘F’ FUNCTION
% THIS FUNCTION OUTPUTS THE GROUND ACCELERATION OF EARTHQUAKE AT TIME t

function accl = f(t)

% HARMONIC FUNCTION EARTHQUAKE
% accl = 10*sin(10*t);

% EL CENTRO EARTHQUAKE
g = 9.814; %Accleration due to gravity, m/sec2.
time = 0:0.02:49.98;
accl = g*spline(time,elcentro_data,t); %Ground acceleration

% THIS IS THE END OF 'F' FUNCTION---------------------------------------------------------------------------


% ‘ELCENTRO_DATA’ FUNCTION
% THIS FUNCTION CONTAINS EL CENTRO EARTHQUAKE DATA.
%
% THE DATA IS IN A NON DIMENSIONAL FORM OBTAINED BY DIVIDING
% BY ACCELERATION DUE TO GRAVITY.
%
% Total readings  First reading    Delta t
%     2500            1              .02

function y = elcentro_data

y= [

EL CENTRO EARTHQUAKE DATA

];

% THIS IS THE END OF ‘ELCENTRO_DATA’ FUNCTION---------------------------------------------------
```

VITA

Vivek K. Agarwal was born on 10th January 1979 at Bikaner in Rajasthan, India. After graduating from high school and appearing in national competitive examinations, he was accepted in the fall of 1998 to Indian Institute of Technology, Delhi. From there he graduated with a degree of Bachelor of Technology in civil engineering in the spring of 2002. In the fall of the same year he came to Texas A&M University and earned a degree of Master of Science in civil engineering in the spring of 2004. The focus of studies for his Master's degree was structural engineering. While a graduate student, he also worked as a Graduate Research Assistant from the fall of 2002 to the spring of 2004.

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